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Derivation and Evaluation of an Adaptive PID Controller

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled Derivation and Evaluation of an Adaptive PID Controller submitted by Supardi Tjokro in partial fulfilment of the requirements for the degree of Master of Science in Process Control.

To My Family

Abstract

Conventional PID controllers are used commonly in industry. However, under various circumstances such as changing process conditions, the PID loop does not necessarily perform in an 'optimal' manner, and retuning may then be required. Considering the large number of PID loops in industrial plants, an adaptive scheme, that allows the PID loops to be automatically tuned to accomodate changes in the process and/or the environment, is highly desirable.

This thesis derives an adaptive PID(PI) controller from the synthesis of classical pole-placement design technique and modern control theory. The resulting controller has the following features:

1. It is structurally and mathematically equivalent to a conventional discrete PID(PI) controller.
2. It ensures both asymptotic servo and regulatory properties of the resulting closed-loop system in the presence of setpoint changes, noise and/or unmeasurable sustained load disturbances.
3. It allows the use of adaptive feedforward control to improve the closed-loop regulatory response in the presence of measurable disturbances.
4. It can handle unknown and constant or varying time delay systems.
5. It is algorithmically implicit.
6. Its parameter estimation routine combines the

computationally efficient and numerically stable U-D factorization method with a variable forgetting factor.

7. It provides two on-line tuning parameters: a desired closed-loop pole location, and a value to form the variable forgetting factor.

The algorithm is implemented on a HP-1000 digital computer to control the temperature of a stirred-tank heater, which has a variable time delay. The evaluations focus mainly on the initial parameters to start the algorithm, and the controller parameters. Results from both simulation and experimentation show the superior performance of the adaptive PID(PI) controller as compared to the fixed gain PID(PI) controller. The adaptive PID(PI) controller is therefore a logical alternative to the fixed gain PID(PI) controller, particularly when retuning is often required.

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Nomenclature

$A(z^{-1})$	Process output polynomial.
$\bar{B}(z^{-1})$	Process input polynomial.
$B(z^{-1})$	Process input polynomial which includes extra terms to model the corresponding time delay.
$e(t)$	*A posteriori* estimation error.
$E_n(z^{-1}),E_d(z^{-1})$	Feedforward controller polynomials.
$F(z^{-1}),G(z^{-1}),H(z^{-1})$	Feedback controller polynomials.
$\mathbf{I}$	Identity matrix.
k	Discrete time interval.
Kc	Controller gain.
K_i	Reset time constant ($=1/T_i$).
$\bar{L}(z^{-1})$	Measurable disturbance polynomial.
$L(z^{-1})$	Measurable disturbance polynomial which includes extra terms to model the corresponding time delay.
N_0	The nominal asymptotic memory length of the estimator.
nt	Total number of parameters to be estimated.
$\mathbf{P}(t)$	Covariance matrix of the parameter estimator.
$S(z^{-1})$	Desired closed-loop zeros polynomial corresponding to the unmeasured noise and/or disturbance.
T_d	Derivative constant.

T_i	Integral constant.
T_s	Sampling interval.
$u(t)$	Process input from feedback path.
$u_1(t)$	The sum of process input from feedback and feedforward paths.
$u_2(t)$	Process input from feedforward path.
$v(t)$	Measurable disturbance.
$W(z^{-1})$	Desired closed-loop characteristic polynomial.
$X(z^{-1})$	Desired closed-loop zeros polynomial corresponding to the process setpoint.
$y(t)$	Process output.
$y_s(t)$	Process setpoint.
$\theta(t)$	Parameter vector.
$\kappa(t)$	Parameter estimator gain.
μ	Forgetting factor.
$\xi(t)$	Unmeasurable noise and/or disturbance.
σ^2	Noise variance.
Σ_0	Parameter defined as $\sigma^2 N_0$.
τ	Process time constant.
$\psi(t)$	Input-output vector.

Superscript

^{d}	Discrete time delay of process input written in terms of sampling period.
^{i}	Order of polynomial $\bar{L}(z^{-1})$.
^{j}	Discrete time delay of process

	disturbance written in terms of sampling period.
m	Order of polynomial $\bar{B}(z^{-1})$.
m_i	Order of polynomial $W(z^{-1})$.
m_j	Order of polynomial $X(z^{-1})$.
m_k	Order of polynomial $S(z^{-1})$.
n	Order of polynomial $A(z^{-1})$.
n_i	Order of polynomial $F(z^{-1})$.
n_j	Order of polynomial $G(z^{-1})$.
n_k	Order of polynomial $H(z^{-1})$.
t	Vector transpose.
^	Estimated value.

Figure title

Da	Actual time delay.
De	Number of extra parameters in polynomial $\hat{B}(z^{-1})$.
G	Process gain.
M	Position of the non-zero initial parameter in the parameter vector counted from $\hat{a}_1$.
Po	Initial value of the covariance matrix.
So	Is equivalent to Σ_0.
W	Is equivalent to the desired closed-loop pole location w_1.
w.ffd.	With feedforward compensation.
w.o.ffd.	Without feedforward compensation.

1. Introduction

1.1 Introduction

When the system model is known, a vast array of standard design techniques can be employed to generate different control strategies. Often, however, the system model is only partially known or even completely unknown. The design techniques then may incorporate some form of on-line parameter estimation technique. This leads to adaptive, or self-tuning control schemes.

With an adaptive or self-tuning algorithm, the controller parameters are continually updated from the estimates of the model parameters. An adaptive control system is therefore defined as a control system which can adjust its controller settings automatically, in such a way as to accomodate changes in the process it controls or its environment.

This chapter briefly outlines the historical development of adaptive controllers, the different adaptive design techniques, and the developments in adaptive PID control.

1.2 Adaptive Control

The concept of adaptive control system was first introduced in the late 1950's. The idea of model reference adaptive control (MRAC), for instance, was originally proposed by Whitaker, Yamron and Kezer [1958] and that of

self-tuning regulator (STR) by Kalman [1958]. Classical linear controllers were found to give satisfactory performance only at a single operating condition. Motivated mainly by the design of autopilots for high performance aircraft and rockets over a wide range of flight conditions, research on adaptive control was very active in the late 1950's. This enthusiasm soon diminished due to poor hardware and insufficient theory to fully analyze and understand such systems.

During the 1960's many new control theories were introduced. Among them were state-space based methods such as linear quadratic gaussian (LQG) techniques, stability theory, stochastic control theory, system identification and parameter estimation. Development of these theories has led to a better understanding of the design and operation principles of adaptive control systems, and provided the backbone to some of the more recent developments in adaptive control.

Revived interest in adaptive control, however, did not begin until the major step forward made by Åström and Wittenmark [1973] in self-tuning control theory, and the availability of inexpensive computer control hardware. Since then many adaptive controllers have been developed [Landau, 1973; Martin-Sanchez, 1974; Clarke and Gawthrop, 1975; Goodwin, Radmage and Caines, 1978; Hoopes, Hawk and Lewis, 1982; Hawk, 1983]. Recent surveys on the subject include those by Seborg, Shah and Edgar [1983] and Åström [1983].

1.3 Design of Adaptive Control Systems

The design of adaptive control systems can be classified into two basic categories:

1. Explicit or indirect method.
2. Implicit or direct method.

This type of classification is convenient and has been frequently used in the literature [Narendra and Valavani, 1979; Seborg, Shah and Edgar, 1983]. However, it is not as clearcut as may first appear. The two categories have been shown, from stability analysis, to be similar and in some cases identical [Ljung, 1978; Egardt, 1979].

In the explicit method, parameters of the process model with a pre-specified structure are estimated on-line recursively using the process input-output history. Design calculations of the control law are then based on the estimated model parameters. It is called explicit since the process is identified explicitly and the identified model parameters are used 'indirectly' to design the control law. It is clear, then, that as long as estimates of the model parameters are available, almost any method can be used to design the control law. The explicit method is therefore flexible with respect to control law design. A potential shortcoming of this approach is that the design calculations of the control law may require more computational time than that for the implicit method.

In the direct approach the system is parameterized in such a way that the estimated parameters are also the

controller parameters, i.e. they are used directly in the control law. Design calculations of the control law are then not required.

The different adaptive control schemes proposed in the literature can also be broadly classified into three main categories:

1. Those designed using stability theory, e.g. model reference adaptive control.
2. Those designed by minimizing a quadratic cost function, e.g. self-tuning regulators and controllers.
3. Those designed from a pole-zero placement approach [Wellstead, Edmunds, Prager and Zanker, 1979; Åström and Wittenmark, 1980].

However, there is structural as well as mathematical equivalence between many of these algorithms. For example, a model reference adaptive controller and self-tuning controller have been shown to have structural and algorithmic similarities [Ljung and Landau, 1978; Landau, 1979; Shah and Fisher, 1980; Egardt, 1980]. Connections between self-tuning controller and pole-placement controller have also been demonstrated [Gawthrop, 1977].

Despite the relatively mature state of adaptive control theory, few actual implementations of adaptive controllers in industrial type environments have been reported. One of the possible reasons for the lack of industrial applications may be the seemingly complicated structure of adaptive

controllers, as well as the unfamiliar theory of on-line identification and control. This has led to the development of adaptive or self-tuning PID controllers, which it is hoped will serve as replacements for the commonly used conventional discrete PID controllers for troublesome processes, and as a stepping-stone for the use of more complicated adaptive controllers.

1.4 Adaptive PID Controller

Conventional feedback control systems typically include controllers of the well-known PID type. This type of controller, when properly tuned to the process, performs very well. Many tuning schemes have been reported in the literature for tuning PID controller settings. Yet a large percentage of PID controllers in industrial plants are still found to operate in manual mode and badly tuned [Andreiev, 1981]. There are three main reasons for this. First, most tuning techniques are based on optimization theory, e.g. Fletcher-Powell method with a quadratic control performance criteria [Isermann, 1981]. The tuning procedure can therefore be rather tedious and time consuming, especially when retuning is frequently needed.

Second, most chemical processes are non-linear in nature. The linearized models that are used to design classical linear PID controllers depend, therefore, upon the particular steady-state operating conditions around which the processes are linearized. It is obvious then that if the

process operating condition changes significantly, the controller settings must be retuned to give satisfactory results. Third, chemical processes are often non-stationary, e.g. process dynamics vary with a change in the process. For example, decay or ageing of catalyst activity in a reactor would require retuning of the controller settings, time and again, to achieve the desired closed-loop performance. Considering the large number of control loops in industrial plants, it is clear that adaptation of the controller settings to accomodate changes in the process and/or its environment is highly desirable.

In the past decade, the dramatic progress in computer technology has been accompanied by an extensive research effort in the development of adaptive control algorithms. Yet a recent review of applications of adaptive controllers [Parks, Schaufelberger, Schmid and Unbehauen, 1980] showed rather discouraging results and stated that, "the number of significant applications is really quite small ... reports on applications are thin and performance data from such systems even thinner". On the other hand, the level of closed-loop control done by digital computers has increased enormously due to the rapid fall in the cost of computer hardware. Most of them, however, are done by using very short sample times to 'mimic' the conventional analog PID controllers for several reasons:

1. They are the best understood, easiest to implement and maintain.

2. They are 'robust' and effective for a wide range of applications.
3. Industrial processes are often strongly non-linear and tend to change in an unpredictable way.
4. In practice, it is often difficult to determine *a priori* economic benefits for the applications of modern control.

Although it has been demonstrated that a great advantage can be gained in some cases by using advanced multivariable controllers, there are many more cases in which improvements over fixed gain PID controllers can be obtained by using adaptive PID algorithms to compensate for changes in the controlled processes and the accompanying dead times.

The presence of time delay in the controlled process is a typical characteristic of many process control problems. The existence of this time delay can greatly complicate the analytical design aspect of the control system. Moreover, time delay adds pure phase lag and thus reduces the stability of the closed-loop system. To compensate for this, the controller gain must be reduced from the one which would be used for the same process without delay. Consequently, time delay also limits the achievable system performance.

One of the most widely used time delay compensation techniques is the Smith Predictor control scheme [Smith, 1957], since it can be used with any conventional regulator of the PID type. To design this controller, it is essential

to have a fairly accurate model of the process. However, perfect modeling is difficult to achieve in practice, if not impossible, due to process parameter variations. Any mismatch in the model causes loss of performance. This loss in performance can be reduced by including an adaptive mechanism to continually estimate the process model parameters and using these parameters to update the time delay compensator. This approach is known as the certainty-equivalence principle in which the controller is designed assuming that the estimated process model is the actual process itself.

Time delay in chemical processes often appears as transport delay and varies with the process flowrate. Because time delay is a difficult parameter to be estimated on-line [White, 1976], Vogel [1982] proposed a dead-time compensator/controller which can adapt to unknown or varying time delay systems. Though it was developed independently, Vogel's dead-time compensator/controller can be shown to be a particular case of Åström's design based on pole-zero placement technique [Seborg, Shah and Edgar, 1983].

Design of adaptive controllers based on pole-zero placement method has been considered by many authors in the literature. In particular this idea has been popularized by Wellstead and his co-workers [Wellstead, Edmunds, Prager and Zanker, 1979; Wellstead, Prager and Zanker, 1979]. Their work focused mainly on the regulation problem. The use of feedforward control for measurable disturbances was not

discussed. In addition, their algorithm did not guarantee setpoint tracking. The use of feedforwarding the setpoint for tracking was then included in the original pole-placement regulator [Wellstead and Zanker, 1979]. The resulting algorithm is capable of handling both servo and regulation problems at the expense of more computational effort. With the same aim, an extended pole-placement self-tuning algorithm was later proposed by Wellstead and Sanoff [1981].

Wouters [1977] proposed a stochastic pole-placement strategy. Again, his emphasis was on the stochastic regulation problem. The self-tuning controller introduced by Clarke and Gawthrop [1975 and 1979], as discussed by Gawthrop [1977], can be interpreted in a pole-placement framework. Åström and Wittenmark [1980] also proposed a general procedure to design pole-zero placement based self-tuning controllers. Their work, however, focused entirely on the servo problem.

Generally, adaptive controller design based on the quadratic cost function approach, e.g. self-tuning controller, or those based on pole-zero placement techniques do not have the structure of a conventional PID controller, especially for systems with time delays. Generating good PID controller settings from process parameter estimations was reported to be difficult [Vogel and Edgar, 1980]. Despite many papers in this area, the presence of a varying time delay in a process for the design of an adaptive PID

controller has either been ignored or was treated as known and constant. Design of simple self-tuning controllers based on the pole-placement method was discussed by Wittenmark and Åström [1980] and Isermann [1981]. The algorithm of Wittenmark and Åström leads to a three-mode action controller and contains a memory of the previous control actions. Calculations of the three PID controller settings are not explicit , and therefore the three-mode action controller can be regarded as a generalized PID controller. An overview of these generalized PID algorithms can be found in the work by Harris, MacGregor and Wright [1982]. While Isermann's algorithm provides explicit equations to calculate the PID settings, the derivation was based on a pre-specified controller transfer function. By so doing, all the process zeros are forced to the origin of the z-plane except one at z=1 to form an integral action controller. Based on phase and amplitude margins specification, Åström and Hägglund [1983] also developed an automatic tuning procedure for tuning of conventional PID regulators. Self-tuning of PID controller settings has also been discussed by Corripio and Tompkins [1981]. They combined the instrumental variable parameter estimation technique proposed by Touchstone [1975] and the Dahlin digital control tuning method [Dahlin, 1968] to generate the adaptive algorithm. Moreover, the second parameter in the numerator polynomial of a second order process, i.e. b_2, is forced to zero in order to obtain explicit equations for the PID

settings.

Combining a continuous time process model and a discrete time adaptive controller, Gawthrop [1980] developed a hybrid self-tuner. Using this hybrid approach, Gawthrop [1981] also derived a self-tuning PI(PID) controller which enables the controller parameters in continuous form to be tuned by a discrete time estimator. This approach offers an advantage of avoiding non-minimum phase characteristic caused by discretization of the continuous process due to certain choice of sampling time or fractional part of a time delay. Bànyàsz and Keviczky [1982] also suggested an adaptive scheme to calculate the PID constants by using a gradient search method to ensure a pre-determined overshoot for the closed-loop step response. The scheme was recently extended to include a correcting term for non-minimum phase systems [Hetthéssy, Keviczky and Bànyàsz, 1983].

Cameron and Seborg [1983] presented a self-tuning PID controller based on a modified version of Clarke and Gawthrop's self-tuning controller. The resulting PID controller has its proportional and derivative modes act on the filtered measurement and integral mode on the control error. Moreover, integral action is introduced by forcing the dynamics of the input variable. More recently a robust self-tuning feedback controller has been developed, which guarantees global convergence of the control error in the presence of bounded noise and/or unmeasured disturbances. In its simplest form, the self-tuning feedback controller is

mathematically and structurally equivalent to the discrete PID algorithm [Song, 1983; Song, Shah and Fisher, 1983].

Adaptive PID controllers have also appeared recently as commercial products in North America, Japan and Europe. For example, Leeds & Northrup Company introduced a self-tuning PID option into their new Electromax V single-loop controller [Andreiev, 1981]. Toshiba has also developed a controller which is capable of deciding the optimal PID constants automatically. Dedicated systems for specific applications such as cement kilns are also available [Seborg, Shah and Edgar, 1983].

The main theme of this work is to develop an adaptive controller with PID structure to handle unknown and/or varying time delay systems. The derivation includes synthesis of the classical linear feedback control theory, i.e. pole-zero placement, and results of the modern control theory. The major distinctions of this work from the previous ones are:

1. It presents a unified design approach, in a classical linear feedback control theory framework, by considering both the servo problem and the regulatory problem.
2. It allows the use of adaptive feedforward control to improve the closed-loop system performance in the presence of measurable disturbances.
3. The resulting PID controller is capable of

handling unknown and/or varying time delay systems.

4. The controller has integral action which arises naturally from the assumption that the desired closed-loop characteristic equation can be represented by a first order polynomial.
5. The algorithm is implicit since the estimated parameters are used directly as the controller parameters.
6. To avoid numerical stability problems with the recursive computation of the covariance matrix, this study uses the Upper-Diagonal factorization method of Bierman [1976] to estimate the process parameters. In addition this study uses a variable forgetting factor to prevent 'blow-up' of the covariance matrix, and to control the speed of adaptation due to changes in the process and/or the environment.

Furthermore, the adaptive PID controller proposed here can be used in two different ways:

1. As an adaptive controller which is tuned on-line automatically.
2. As a retuning algorithm for the conventional digital PID controllers as the controlled process changes.

1.5 Organization of Thesis

Chapter 2 formulates the design problem and the necessary theory for the development of an adaptive PID controller based on pole-placement method. An adaptive feedforward compensation scheme is also included as an option to improve the system performance due to measurable disturbances. Theoretical derivations of the adaptive PID controller and adaptive feedforward compensator in Chapter 2 assume that the process parameters are known. For processes with unknown and/or slowly changing parameters, an on-line recursive scheme is required to estimate the process parameters and update the controller settings accordingly. Such a scheme is investigated in Chapter 3. Chapter 4 discusses the results of implementing the proposed adaptive PID controller on a simulated model. Experimental evaluations of the adaptive PID controller with and without the adaptive feedforward compensator on a stirred-tank heater are presented in Chapter 5, followed by conclusions and recommendations for future work in Chapter 6.

2. Adaptive Pole-Placement Controller

2.1 Introduction

Adaptive control techniques were introduced to improve controller performance and, above all, to eliminate the time-consuming manual tuning procedure of conventional PID controllers. However, adaptive controllers also have tuning parameters and they are oftentimes more difficult to tune or initialize than PID controllers.

Based on these facts, the objective of this study was to develop an adaptive controller which is structurally and mathematically equivalent to a conventional discrete PID algorithm, and whose tuning parameters are easier to choose. Other considerations in this development are: (i) to ensure the applicability of the resulting controller to systems with unknown but constant or varying time delays and (ii) to ensure asymptotic closed-loop tracking and regulation.

The following section presents a general approach to the design of a pole-placement controller. Simplification and refinement of this approach leading to the formulation of an adaptive PID controller is given in section 2.3. Since it is a well-known result that feedforward compensation provides good disturbance rejection, the design of an adaptive feedforward compensator when process disturbances are measurable is considered in section 2.4.

2.2 Theory

Consider a single-input single-output process which can be characterized by a linear, discrete ARMA model:

$$y(k) = \frac{z^{-d-1}\bar{B}(z^{-1})}{A(z^{-1})} u(k) + \frac{z^{-j-1}\bar{L}(z^{-1})}{A(z^{-1})} v(k) + \xi(k) \quad (2.1)$$

where u(k), y(k) and v(k) denote the process input, output and measurable deterministic disturbance respectively. $\xi(k)$ is the residual which accounts for the effects of unmeasured disturbances, process and/or measurement noise, modeling errors, process non-linearities etc.. d and j are the corresponding time delays written in terms of sampling intervals. z^{-1} is the backward shift operator, i.e. $z^{-1}y(k) = y(k-1)$, and the polynomials $A(z^{-1})$, $\bar{B}(z^{-1})$ and $\bar{L}(z^{-1})$ are defined as:

$$\begin{aligned} A(z^{-1}) &= 1 + a_1 z^{-1} + \ldots + a_n z^{-n} \\ \bar{B}(z^{-1}) &= \bar{b}_1 + \bar{b}_2 z^{-1} + \ldots + \bar{b}_m z^{-m} \\ \bar{L}(z^{-1}) &= \bar{l}_1 + \bar{l}_2 z^{-1} + \ldots + \bar{l}_i z^{-i} \end{aligned} \quad (2.2)$$

However, equation (2.1) can be rewritten in a more compact form as:

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})} u(k) + \frac{L(z^{-1})}{A(z^{-1})} v(k) + \xi(k) \quad (2.3)$$

where $B(z^{-1}) = z^{-1}(b_1 + b_2z^{-1} + \ldots + b_{m+d}z^{-m-d+1})$

$$L(z^{-1}) = z^{-1}(l_1 + l_2z^{-1} + \ldots + l_{i+j}z^{-i-j+1}) \quad (2.4)$$

This system description does not require an explicit estimation of the time delay and allows a maximum time delay of d and j sampling intervals to be considered in the process and the disturbance polynomials respectively [Wellstead and Sanoff, 1981]. Ideally, the leading d and j coefficients of the corresponding polynomials $B(z^{-1})$ and $L(z^{-1})$ will be zero or close to zero for time delays of d and j sampling intervals.

In the derivation of the adaptive controller the measurable disturbance term is initially omitted for simplicity. Equation (2.3) then becomes:

$$y(k) = \frac{B(z^{-1})}{A(z^{-1})} u(k) + \xi(k) \quad (2.5)$$

A general linear feedback controller has the form :

$$u(k) = \frac{H(z^{-1})}{G(z^{-1})} y_s(k) - \frac{F(z^{-1})}{G(z^{-1})} y(k) \quad (2.6)$$

where $y_s(k)$ is the process setpoint; the polynomials $F(z^{-1})$, $G(z^{-1})$ and $H(z^{-1})$ are defined as:

$$F(z^{-1}) = f_1 + f_2 z^{-1} + \ldots + f_{ni} z^{-ni}$$
$$G(z^{-1}) = 1 + g_1 z^{-1} + \ldots + g_{nj} z^{-nj} \tag{2.7}$$
$$H(z^{-1}) = h_1 + h_2 z^{-1} + \ldots + h_{nk} z^{-nk}$$

According to pole-placement criterion, the coefficients of polynomials $F(z^{-1})$, $G(z^{-1})$ and $H(z^{-1})$ are determined in such a way that the desired closed-loop transfer function of the given system can be represented by :

$$y(k) = \frac{X(z^{-1})}{W(z^{-1})} y_s(k) + \frac{S(z^{-1})}{W(z^{-1})} \xi(k) \tag{2.8}$$

The polynomials $X(z^{-1})$, $S(z^{-1})$ and $W(z^{-1})$ are chosen by the designer and defined as :

$$S(z^{-1}) = s_1 + s_2 z^{-1} + \ldots + s_{mk} z^{-mk}$$
$$X(z^{-1}) = x_1 + x_2 z^{-1} + \ldots + x_{mj} z^{-mj} \tag{2.9}$$
$$W(z^{-1}) = 1 + w_1 z^{-1} + \ldots + w_{mi} z^{-mi}$$

such that at steady-state (as $k \to \infty$) $X(z^{-1})/W(z^{-1}) = 1$ and $S(z^{-1})/W(z^{-1}) = 0$.

From Figure 2.1, the closed-loop transfer function relating $y(k)$ to $\xi(k)$ is given by:

$$y(k) = \frac{A(z^{-1})\ G(z^{-1})}{A(z^{-1})\ G(z^{-1}) + B(z^{-1})\ F(z^{-1})} \xi(k) \tag{2.10}$$

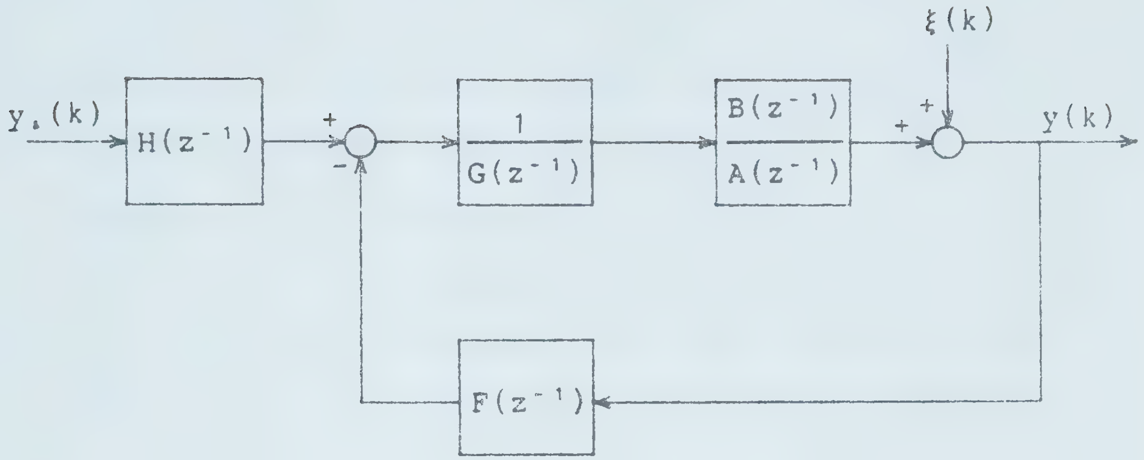

Figure 2.1. Block diagram of a closed-loop system.

and that relating $y(k)$ to $y_s(k)$ can be written as:

$$y(k) = \frac{B(z^{-1})\ H(z^{-1})}{A(z^{-1})\ G(z^{-1}) + B(z^{-1})\ F(z^{-1})}\ y_s(k) \tag{2.11}$$

Equating equation (2.10) and equation (2.11) to the corresponding parts of equation (2.8) results in:

$$A(z^{-1})\ G(z^{-1}) + B(z^{-1})\ F(z^{-1}) = \frac{A(z^{-1})\ G(z^{-1})\ W(z^{-1})}{S(z^{-1})} \tag{2.12}$$

$$H(z^{-1}) = \frac{A(z^{-1})\ G(z^{-1})\ X(z^{-1})}{B(z^{-1})\ S(z^{-1})} \tag{2.13}$$

Now the design problem is reduced to the algebraic solution of equations (2.12) and (2.13) to find the controller polynomials $F(z^{-1})$, $G(z^{-1})$ and $H(z^{-1})$, providing that the polynomials $A(z^{-1})$ and $B(z^{-1})$ are known and polynomials $S(z^{-1})$, $X(z^{-1})$ and $W(z^{-1})$ are pre-specified.

Equation (2.12) displays the heart of a pole-placement approach, i.e. the original closed-loop characteristic equation described by the left hand side of equation (2.12) is replaced by that of the right hand side. From the stability analysis of feedback systems in classical linear feedback control theory, it is a well-known result that the stability characteristic of a feedback system is determined by the pole locations of its closed-loop characteristic equation. Therefore, as long as the roots of the right hand side polynomials are inside the unit circle of the z-plane, the overall feedback system will be stable.

The design procedure described above, however, is not without shortcomings. The resulting algorithm is explicit and its implementation requires three steps at each sampling intervals: i) on-line estimation of the process model parameters, equation (2.5); ii) calculation of the controller parameters from equations (2.12) and (2.13); iii) determination of the control input from equation (2.6). Therefore, adaptive controllers based on this design approach usually require more computational effort, since equations (2.12) and (2.13) have to be solved at each sampling interval for the coefficients of polynomials

$F(z^{-1})$, $G(z^{-1})$ and $H(z^{-1})$. In the sequel, certain procedures will be employed to remove this requirement, i.e. to design a computationally more efficient implicit algorithm, and at the same time derive an adaptive controller with a conventional discrete PID structure.

2.3 Derivation of An Adaptive PID Controller

The question which often arises in pole-placement method is how to choose the polynomials $S(z^{-1})$, $X(z^{-1})$ and $W(z^{-1})$. The answer to this question has not been dealt with explicitly in the literature, and the polynomials $S(z^{-1})$, $X(z^{-1})$ and $W(z^{-1})$ are often referred to as the polynomials describing the 'desired closed-loop system'. In general, it is found that any particular solution can be obtained depending on the design objectives to be achieved. The following treatment demonstrates how this can be done. For the specific case here, the design objectives are:

1. To design a pole-placement controller such that no process zeros are cancelled.
2. To obtain a simple and practical controller such that design effort and computational time are minimum.
3. To obtain a controller with conventional discrete-time PID structure.
4. To have a system with good asymptotic closed-loop tracking property, i.e. $y(k)/y_r(k) \rightarrow 1$ as $k \rightarrow \infty$.

5. To have a system with good asymptotic closed-loop regulatory property, i.e. $y(k)/\xi(k) \rightarrow 0$ as $k \rightarrow \infty$.

6. To be able to handle unknown and/or changing time delay systems.

Now consider the following choice of $S(z^{-1})$:

$$S(z^{-1}) = \frac{G(z^{-1})}{K_1} \tag{2.14}$$

where K_1 is a constant whose value will be determined in the sequel. The reasons for this choice of $S(z^{-1})$ are: i) to simplify the solutions of equations (2.12) and (2.13); ii) to reduce computational time as the determination of $G(z^{-1})$ will also depend on how $S(z^{-1})$ is specified; iii) to obtain asymptotic regulatory control. How $S(z^{-1})$ effects this later objective will become clear in the discussion to follow. Making use of this choice, equations (2.12) and (2.13) reduce to:

$$A(z^{-1})\ G(z^{-1}) + B(z^{-1})\ F(z^{-1}) = K_1\ A(z^{-1})\ W(z^{-1}) \tag{2.15}$$

$$H(z^{-1}) = \frac{K_1\ A(z^{-1})\ X(z^{-1})}{B(z^{-1})} \tag{2.16}$$

Solving for the coefficients of polynomials $F(z^{-1})$ and $G(z^{-1})$ from equation (2.15) yields a set of simultaneous equations. Mathematically, it is also clear that equation

(2.15) is solvable only when the matrix formed by the coefficients of polynomials $A(z^{-1})$ and $B(z^{-1})$ is not singular. An iterative algorithm to solve equation (2.15) asymptotically was proposed by Elliott and Wolovich [1979], and Kreisselmeier [1980]. Their approach, though seems to alleviate the matrix singularity problem, is computationally unattractive in practical applications, and will still encounter the same difficulty when matrix singularity occurs [Goodwin and Sin, 1984]. An alternative is to choose one of two polynomials. Solution for the other will then be unique, and solving of the simultaneous equations and the difficulty due to matrix singularity can be eliminated. Different choices of the polynomials will inevitably lead to different closed-loop properties. In this work, polynomial $F(z^{-1})$ is chosen and given by:

$$F(z^{-1}) = K_2 \, A(z^{-1}) \tag{2.17}$$

where K_2 is a constant and the choice of its value will be explained shortly. This choice of polynomial $F(z^{-1})$ is significant in the outcome of the resulting algorithm. Since $F(z^{-1})$ is the feedback controller polynomial and $A(z^{-1})$ is the process output polynomial, estimation of the process model parameters in an adaptive control context will directly determine the controller parameters. Similarly, the result is extended to polynomial $G(z^{-1})$, as substitution of equation (2.17) into equation (2.15) yields:

$$G(z^{-1}) = K_1\, W(z^{-1}) - K_2\, B(z^{-1}) \tag{2.18}$$

Where $W(z^{-1})$ is user specified and $B(z^{-1})$ is the process input polynomial.

Calculation of the control law, equation (2.6), is now left with the determination of $H(z^{-1})$ from equation (2.16). In order to avoid cancellation of the possibly unstable or poorly damped zeros of the system, the choice of polynomial $X(z^{-1})$ is made such that the desired closed-loop zeros are the process zeros, i.e.

$$X(z^{-1}) = \frac{K_2\, B(z^{-1})}{K_1} \tag{2.19}$$

This choice of $X(z^{-1})$ therefore offers an advantage to the resulting pole-placement controller in handling non-minimum phase systems. However, it should not be assumed that knowledge of the process zeros is required *a priori* since polynomial $B(z^{-1})$ can be estimated on-line in the estimator. Substitution of equation (2.19) into equation (2.16) gives :

$$H(z^{-1}) = K_2\, A(z^{-1}) \tag{2.20}$$

From equations (2.17), (2.18) and (2.20), it is clear that the controller parameters are all expressed in terms of the process parameters. This type of algorithm is called implicit or direct.

The values of K_1 and K_2 can now be determined by considering the design objectives, i.e. the controller should have good asymptotic tracking and regulatory properties. If equations (2.14), (2.18) and (2.19) are substituted into equation (2.8), the following equation will be obtained:

$$y(k) = \frac{K_1\ W(z^{-1}) - K_2\ B(z^{-1})}{K_1\ W(z^{-1})}\ \xi(k) + \frac{K_2\ B(z^{-1})}{K_1\ W(z^{-1})}\ y_s(k) \tag{2.21}$$

Obviously, in order to achieve $y(k)/\xi(k) \rightarrow 0$ and $y(k)/y_s(k) \rightarrow 1$ as $k \rightarrow \infty$, the constants K_1 and K_2 should be:

$$K_1 = \lim_{z^{-1} \rightarrow 1} B(z^{-1}) = \sum_{i=1}^{m+d} b_i \tag{2.22}$$

$$K_2 = \lim_{z^{-1} \rightarrow 1} W(z^{-1}) = \sum_{i=0}^{mi} w_i \tag{2.23}$$

and the control law becomes:

$$u(k) = \frac{\Sigma w_i\ [A(z^{-1})\ y_s(k) - A(z^{-1})\ y(k)]}{\Sigma b_i\ W(z^{-1}) - \Sigma w_i\ B(z^{-1})} \tag{2.24}$$

Providing that polynomials $A(z^{-1})$ and $B(z^{-1})$, process setpoint $y_s(k)$ and process output $y(k)$ are given,

determination of equation (2.24) only requires user specified closed-loop poles, i.e. polynomial $W(z^{-1})$. For high order systems, it may be difficult to specify all the closed-loop poles. Also, it may be impractical to model the process with a high order model from computer storage and computational time viewpoints. In most cases, it is satisfactory to model the process by a second order plus time delay model or a first order plus time delay model, and to specify only the dominant poles.

In practice, it is desirable to have a simple yet effective control law, e.g. PID control law. A PID controller in its velocity form may be written as:

$$\begin{aligned} u(k) = u(k-1) + Kc\{&- y(k) + y(k-1) \\ &+ [y_s(k) - y(k)]T_s/T_i \\ &+ [- y(k) + 2y(k-1) - y(k-2)]T_d/T_s\} \end{aligned} \tag{2.25}$$

Equation (2.25) is one of the various forms of conventional digital PID control law used in literature [Phelan, 1977; Isermann, 1981]. Rearranging this equation gives:

$$\begin{aligned} u(k) = u(k-1) &+ \frac{Kc\ T_s}{T_i} y_s(k) \\ &- Kc\left[1 + \frac{T_s}{T_i} + \frac{T_d}{T_s}\right] y(k) \\ &+ Kc\left[1 + \frac{2T_d}{T_s}\right] y(k-1) - \frac{Kc\ T_d}{T_s} y(k-2) \end{aligned} \tag{2.26}$$

Since the setpoint appears only in the integral term, this particular form offers the advantage of avoiding derivative kick and large proportional action at the time of making a setpoint change.

Equation (2.24) is identical to equation (2.26) if:

1. The process can be modeled by a second order plus time delay model, i.e.

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2}$$

$$B(z^{-1}) = b_1 z^{-1} + \ldots + b_{2+d} z^{-2-d} \tag{2.27}$$

2. The steady-state approximation can be used such that:

$$A(z^{-1})\ y_s(k) = \Sigma a_i\ y_s(k) \tag{2.28}$$

$$B(z^{-1}) = \Sigma b_i \tag{2.29}$$

3. $W(z^{-1})$ can be quantified by a first order polynomial, i.e.

$$W(z^{-1}) = 1 + w_1 z^{-1} \tag{2.30}$$

The last condition makes the order of $(K_1\ A(z^{-1})\ W(z^{-1}))$ in the closed-loop equation (2.15) become less than that of $(A(z^{-1})\ G(z^{-1}) + B(z^{-1})\ F(z^{-1}))$. This implies that there are common factors which cancel. If equation (2.10) is equated to the second part of equation (2.8), and at the same time using the result of equation (2.14), the following equation will be obtained:

$$\frac{A(z^{-1})\ G(z^{-1})}{A(z^{-1})\ G(z^{-1}) + B(z^{-1})\ F(z^{-1})} = \frac{Q(z^{-1})\ G(z^{-1})}{Q(z^{-1})\ K_1\ W(z^{-1})} = \frac{S(z^{-1})}{W(z^{-1})} \tag{2.31}$$

The common factor which cancels is polynomial $Q(z^{-1})$ and the roots of this polynomial are therefore assumed to be in the stable region. Moreover, this common factor $Q(z^{-1})$ can be interpreted as the observer polynomial since the control law in equation (2.6) can be shown to be a combination of an observer and a state feedback law [Åström and Wittenmark, 1980].

Using the three conditions stated above, equation(2.24) becomes:

$$u(k) = u(k-1) + \lambda[\Sigma a_i\ y_s(k) - y(k) - a_1\ y(k-1) - a_2\ y(k-2)] \tag{2.32}$$

$$\text{where } \lambda = -\frac{\Sigma w_i}{w_1\ \Sigma b_i} \tag{2.33}$$

$$\Sigma a_i = 1 + a_1 + a_2 \tag{2.34}$$

$$\Sigma b_i = b_1 + \ldots + b_{2+d} \tag{2.35}$$

$$\Sigma w_i = 1 + w_1 \tag{2.36}$$

The three PID controller settings are generated by comparing the coefficients associated with each term of the variables in equation (2.32) and equation (2.26). They are:

$$Kc = -\lambda(a_1 + 2a_2) \tag{2.37}$$

$$T_d = \frac{\lambda\ a_2\ T_s}{Kc} \tag{2.38}$$

$$T_i = \frac{Kc\ T_s}{\lambda - Kc\ T_d/T_s - Kc} \tag{2.39}$$

Many different techniques have been introduced to add an integrator into the control law. Some of these approaches, however, do not provide zero steady-state offset for sustained load disturbances [Morris, Nazer and Wood, 1981]. It should be noted that equations (2.29) and (2.30) have naturally given rise to an integral action in the control law, equation (2.32). More specifically, substitution of equations (2.29) and (2.30) into equation (2.18) for $G(z^{-1})$ gives:

$$\frac{1}{G(z^{-1})} = -\frac{1}{w_1 \Sigma b_i (1-z^{-1})} \tag{2.40}$$

where a discrete pole at z=1 corresponds to an integrator and gives infinite gain at constant inputs. In addition, equation (2.30) has reduced the user specifications to placing only one dominant pole.

2.4 Derivation of An Adaptive Feedforward Compensator

Feedforward compensation for any measurable disturbances can be incorporated into the adaptive PID control scheme described in the previous section. Consider the following block diagram:

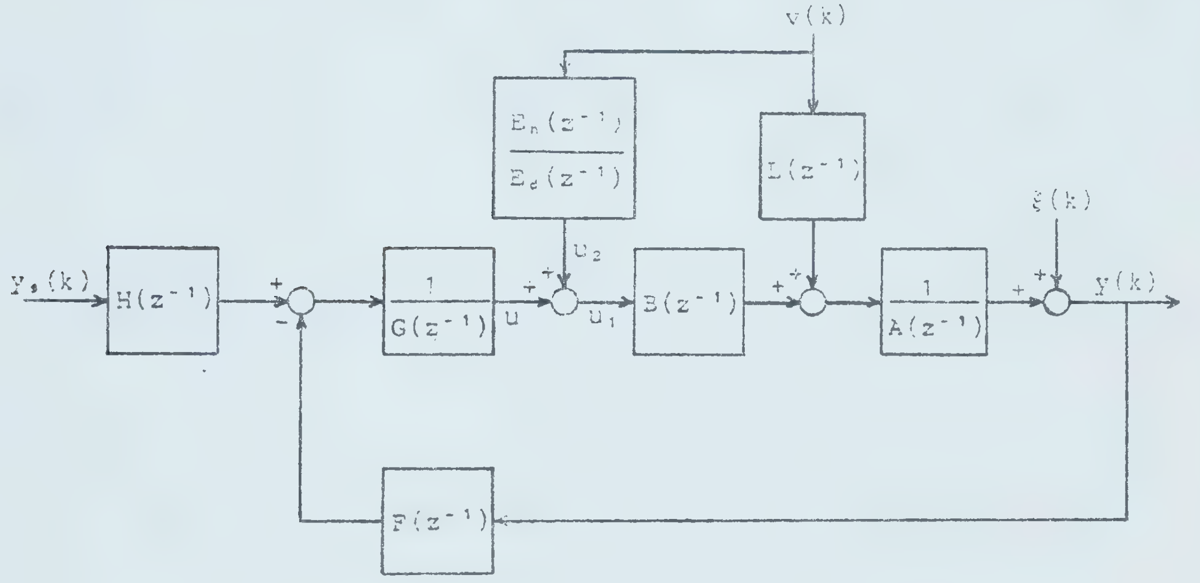

Figure 2.2 Block diagram of a closed-loop system with feedforward compensation.

where $E_n(z^{-1})/E_d(z^{-1})$ is the transfer function of the feedforward compensator that needs to be determined.

The closed-loop transfer function relating y(k) to v(k) is expressed as:

$$y(k) = \frac{G(z^{-1})\left[L(z^{-1}) + \dfrac{E_n(z^{-1})}{E_d(z^{-1})} B(z^{-1})\right]}{A(z^{-1})\, G(z^{-1}) + B(z^{-1})\, F(z^{-1})} v(k) \qquad (2.41)$$

The feedforward compensator is to be designed in such a way that the process output y(k) will be unaffected even in the presence of sustained load disturbances. It is therefore desired to have:

$$G(z^{-1})\left[L(z^{-1}) + \frac{E_n(z^{-1})}{E_d(z^{-1})} B(z^{-1})\right] = 0 \tag{2.42}$$

that is:

$$\frac{E_n(z^{-1})}{E_d(z^{-1})} = - \frac{L(z^{-1})}{B(z^{-1})} \tag{2.43}$$

The feedforward component, u_2, of the total control signal, u_1 (cf. Figure 2.2), is given by:

$$u_2(k) = - \frac{L(z^{-1})}{B(z^{-1})} v(k) \tag{2.44}$$

and the overall control signal is:

$$u_1(k) = u(k) + u_2(k) \tag{2.45}$$

where u(k) is determined from the feedback control law given by equation (2.32).

Since the simplest form of a feedforward compensator is the one with a single gain term, equation (2.44) can be

rewritten by employing the steady-state approximation. This thus reduces to:

$$u_2(k) = - \frac{\Sigma l_i}{\Sigma b_i} v(k) \tag{2.46}$$

The design procedure has been described under the assumption that the process dynamics and environments of the system to be controlled are known, i.e. model parameters are given. For processes with unknown parameters, the model parameters can be simply replaced by their estimated values. This approach is referred to as certainty-equivalence principle in which the estimated model is regarded as the actual process. The design is therefore separated into two steps: identification and control. Estimations of the model parameters can be carried out on-line recursively by any standard method to form the adaptive algorithm. The scheme used in this work is presented in the following chapter.

2.5 Summary

This chapter has described a general approach to pole-placement design, and in the same vein derived an implicit PID(PI) control algorithm. With an on-line estimation routine, the algorithm can be used as a 'stand-alone' adaptive controller, or a retuning algorithm for the existing conventional digital PID(PI) controllers, since the controller settings can be explicitly determined.

In addition, it includes an on-line tuning parameter, w_1, to allow the operator to provide the controller with a knowledge of the desired closed-loop pole location. The last section presents an optional, simple adaptive feedforward compensator to be used with the adaptive PID(PI) algorithm in the presence of a measurable disturbance.

3. Parameter Estimation

3.1 Introduction

In most practical problems there is seldom sufficient *a priori* knowledge about a system and its environment to design a control system. In automatic control, however, it is often possible to identify the system and obtain the missing information by performing experiments on the system. Nevertheless, there are usually severe limitations on the experiments that can be performed in practice. The experiments are often required to be performed during 'normal' operations in order to get useful results. This implies that pertubations on the system, if any, must be small such that the overall production line is undisturbed. Extra regulators might be needed to keep the process fluctuations within the acceptable limits and these in turn might create an influence on the estimation results.

Even in classical control theory for the design of fixed gain PID controllers, some methods of system identification are required, e.g. Bode diagram or Nyquist stability analysis. This chapter deals particularly with the identification scheme to be used with the control strategy presented in Chapter 2 to formulate the adaptive algorithm. For parameter estimation purposes, the system model described by equation (2.3) is rewritten as:

$$\hat{y}(k) = -\hat{a}_1 y(k-1) - \hat{a}_2 y(k-2) + \hat{b}_1 u(k-1) + \ldots + \hat{b}_{m+d} u(k-m-d) + \hat{l}_1 v(k-1) + \ldots + \hat{l}_{i+j} v(k-i-j) \qquad (3.1)$$

The parameters $\hat{a}_1$, $\hat{a}_2$, $\hat{b}_1$, ..., $\hat{b}_{m+d}$, and $\hat{l}_1$, ..., $\hat{l}_{i+j}$ (if feedforward compensation is included) are to be estimated from the process input, output and measurable disturbance histories. $\hat{y}(k)$ is the estimated output of the process $y(k)$.

In vector forms, equation (3.1) can be written compactly as:

$$\hat{y}(k) = \psi^t(k-1)\,\theta(k-1) \qquad (3.2)$$

where
$$\psi^t(k) = [-y(k), \ldots, -y(k-n+1); u(k), \ldots, u(k-m-d+1); v(k), \ldots, v(k-i-j+1)]$$

$$\theta^t(k) = [\hat{a}_1(k), \ldots, \hat{a}_n(k); \hat{b}_1(k), \ldots, \hat{b}_{m+d}(k); \hat{l}_1(k), \ldots, \hat{l}_{i+j}(k)] \qquad (3.3)$$

During the past two decades the field of system identification and parameter estimation has developed rapidly due to its broad applicability in many different areas, adaptive control in particular. There are a number of different schemes proposed in the literature that can be used to perform parameter estimation for the type of process model described by equation (3.3). A few of these are:

- Recursive learning method (RL)
- Recursive least-squares method (RLS)
- Recursive instrumental variable method (RIV)
- Recursive extended least-squares method (ELS)
- Recursive generalized least-squares method (GLS)
- Recursive maximum likelihood method (RML)
- Recursive square-root method (RSQR)
- Recursive upper-diagonal factorization method (RUD)

An overview of the theoretical backgrounds and comparative studies of these schemes can be found in the literature [Nagumo and Noda, 1967; Eykhoff, 1974; Bierman, 1976; Söderström, Ljung and Gustavsson, 1978; Ljung, 1977a, 1977b, 1981; Isermann, 1981; Wong, Bayoumi and Nuyan, 1983]. This work uses a recursive least-squares method with U-D factorization algorithm and a variable forgetting factor. Simulation and experimental results show the success of this combination. As a matter of fact, the recent comparative study by Wong, Bayoumi and Nuyan [1983] shared this same conclusion and stated that, "RUD factorization combined with a variable forgetting factor possesses superior properties ... RSQR and SQR with householder transformation methods may cause parameter drifting ... conventional RLS method should only be used with caution in identification and control especially when the forgetting factor is less than one". To provide further appreciation of this choice, a discussion including illustrated examples is presented in the following

sections.

3.2 Recursive Least-Squares Method

Among the various recursive identification methods the conventional Kalman filtering, or better known as recursive least-squares estimation, is the most popular due to its compact representation, computational efficiency and frequent appearance in the literature. Together with equation (3.2) the recursive least-squares algorithm is given as:

Predicted output: $\hat{y}(k) = \psi^t(k-1)\,\theta(k-1)$ (3.4)

Gain calculation: $$\kappa(k) = \frac{\mathbf{P}(k-1)\,\psi(k-1)}{[1 + \psi^t(k-1)\,\mathbf{P}(k-1)\,\psi(k-1)]} \qquad (3.5)$$

Parameter estimation: $\theta(k) = \theta(k-1) + \kappa(k)[y(k)-\hat{y}(k)]$ (3.6)

Covariance update: $\mathbf{P}(k) = [\mathbf{I} - \kappa(k)\,\psi^t(k-1)]\mathbf{P}(k-1)$ (3.7)

where $y(k)$ and $\hat{y}(k)$ are the process output and the predicted output respectively; $\psi(k)$ is the input-output vector containing the process input, output and disturbance sequences; $\theta(k)$ is a vector containing the parameter estimates and often referred to as parameter vector; $\kappa(k)$ is the estimator gain vector and $\mathbf{P}(k)$ is the covariance matrix.

If the standard recursive least-squares (RLS) method is applied to identify systems with correlated or coloured noise the estimated parameters are biased. This bias can be

removed by applying recursive instrumental variable (RIV), recursive extended least-squares (ELS), recursive generalized least-squares (GLS) or recursive maximum likelihood (RML) at the expense of more computational effort. When RLS is applied to estimate model parameters for adaptive control systems, it is often a question whether the accuracy of the estimates should be judged on the premise of deviations in the model parameters or in the overall system performance. If the final purpose of identification is to design a control system it seems reasonable, then, to judge the accuracy of the estimates on the basis of the overall performance of the control system. Despite its biased estimates, RLS is still the most popular method in the literature because of its superior convergence properties.

Parameter estimation when used in the context of adaptive control has also encountered other problems. For example, system identification and parameter estimation theory assumes persistent excitation of the input signal to the process being considered [Eykhoff, 1974]. This assumption, though seemingly not very crucial, was shown to be one of the conditions necessary for the closed-loop system identifiability [Gustavsson, Ljung and Söderström, 1977; Isermann, 1982] and in some cases, exponential convergence of the estimation and control algorithm [Anderson and Johnson, 1982]. For processes with low noise characteristics such as chemical processes, the input signal is not guaranteed to be persistently excited to give valid

parameter estimates. When the system lacks sufficient excitation for a long period of time, e.g. when the initial excitation has been smoothed out, a phenomenon called parameter drifting may occur. More specifically, the term $\mathbf{P}(k-1)\ \psi(k-1)$ in equation (3.5) will be zero. The gain $\kappa(k)$ in equation (3.5) and hence the term $\kappa(k)\ \psi^t(k-1)$ in equation (3.7) will also be zero. As a result, the parameter estimation vector $\theta(k)$ remains unchanged even if the prediction error, i.e. $y(k) - \hat{y}(k)$, is large. Similarly, there will be no change in the covariance matrix $\mathbf{P}(k)$ in equation (3.7), i.e. $\mathbf{P}(k) = \mathbf{P}(k-1)$.

Example 3.1

Consider a second order system with poles at -0.33 and -0.2 in the s-plane described by the continuous-time transfer function:

$$\frac{y(s)}{u(s)} = \frac{1}{(3s+1)\ (5s+1)} \tag{3.8}$$

Sampling this system with a sampling time of one, the discrete-time input-output relation along with a zero order hold is:

$$\begin{aligned} y(k) = {} & 1.5352\ y(k-1) - 0.5866\ y(k-2) \\ & +0.028\ u(k-1) + 0.0234\ u(k-2) \end{aligned} \tag{3.9}$$

This same model with a time delay ranging from 1 to 5 will again be used as an illustrative model in Chapter 4.

The adaptive PID control law was combined with RLS to control this system to track a unit setpoint trajectory. The setpoint change was made at k=50 and the length of simulation was 700 iterations. The covariance matrix and the gain were initialized to $10^6 I$ and zero respectively. Simulation results at different intervals are shown in the table below:

Table 3.1. List of Covariance Matrix and Gain Using RLS

Iteration #	Covariance Matrix, P(k)	Gain, κ(k)
50	$10^5 \begin{bmatrix} 4.66 & -4.19 & -0.36 & 0.85 \\ -4.19 & 3.84 & 0.34 & -0.74 \\ -0.36 & 0.34 & 4.05 & -3.74 \\ 0.85 & -0.74 & -3.74 & 3.56 \end{bmatrix}$	$\begin{bmatrix} 5.31 \\ -7.06 \\ 0.38 \\ -0.57 \end{bmatrix}$
350	$10^3 \begin{bmatrix} 1.68 & -1.47 & -0.20 & 0.41 \\ -1.47 & 1.29 & 0.17 & -0.35 \\ -0.20 & 0.17 & 0.06 & -0.09 \\ 0.41 & -0.35 & -0.09 & 0.15 \end{bmatrix}$	$\begin{bmatrix} 0.40 \\ -0.36 \\ -0.02 \\ 0.06 \end{bmatrix}$
700	$10^3 \begin{bmatrix} 1.66 & -1.45 & -0.20 & 0.40 \\ -1.45 & 1.27 & 0.17 & -0.35 \\ -0.20 & 0.17 & 0.06 & -0.09 \\ 0.40 & -0.34 & -0.09 & 0.14 \end{bmatrix}$	$\begin{bmatrix} 0.10 \\ -0.09 \\ -0.01 \\ 0.02 \end{bmatrix}$

It is clearly shown in Table 3.1 that the covariance matrix and the gain decrease gradually from k=50 to k=350. Moreover, as time increases the covariance matrix remains

relatively constant while the gain reduces to a value close to zero as expected. This situation is not very desirable in a 'real' system even though the output variable may be tracking the desired trajectory very well. If there is any estimation error due to disturbance, changing time delay, changing process gain etc. in the system, the estimated parameters will not be updated because of the small gain. Consequently, the controller settings which are basically based on the values of parameter estimates will remain unchanged. Under this circumstance the controller has lost its primary purpose of adapting to the process and tuning its settings automatically. In the worst case, the system can even go unstable. It has been suggested in the literature that this situation can be avoided by introducing an additional perturbation signal, or by using a forgetting factor to inflate the covariance matrix.

3.3 Forgetting Factor

If a forgetting factor is introduced into RLS to discount past data when performing the estimations, Equation (3.7) can be rewritten as:

$$\mathbf{P}(k) = \frac{1}{\mu}[\mathbf{I} - \kappa(k)\ \psi^t(k-1)]\mathbf{P}(k-1) \tag{3.10}$$

where μ is the forgetting factor and has a value ranging from 0 to 1.

In practice, the use of this forgetting factor offers additional advantages such as:

1. Since most chemical processes exhibit time-varying and non-linear dynamics which violate the design assumptions of linear and time-invariant systems, a forgetting factor of less than 1 discounts past data and prevents the recursive estimator from converging such that it is able to follow changes in the system due to time-varying and/or non-linear characteristics.
2. The capability to control the speed of adaptation.

However, the choice of this constant value forgetting factor can be critical. It has been reported in the literature [Morris, Fenton and Nazer, 1977; Åström and Wittenmark, 1980; Fortescue, Kershenbaum and Ydstie, 1981] that if this value is not chosen carefully, it can lead to an exponential growth of the covariance matrix and a system which is extremely sensitive to disturbances.

Example 3.2

The same system described in example 3.1 was again simulated with μ=0.8. This value of forgetting factor was chosen to induce faster blow-up phenomenon. The covariance matrix and the gain were initialized to $10^6 I$ and zero respectively. Simulation results are given in Table 3.2.

Table 3.2. List of Covariance Matrix and Gain Using RLS With Constant Forgetting Factor

Iteration #	Covariance Matrix, P(k)	Gain, $\kappa(k)$
1	$10^6 I$	0
50	$10^{10}\begin{bmatrix} 1.04 & -0.89 & -0.04 & 0.21 \\ -0.89 & 0.76 & 0.03 & -0.17 \\ -0.04 & 0.03 & 1.44 & -1.32 \\ 0.21 & -0.17 & -1.32 & 1.24 \end{bmatrix}$	$\begin{bmatrix} 5961 \\ -5507 \\ -1177 \\ 1460 \end{bmatrix}$
65	$10^5\begin{bmatrix} 1.47 & -1.33 & -0.23 & 0.40 \\ -1.33 & 1.20 & 0.21 & -0.36 \\ -0.23 & 0.21 & 0.05 & -0.08 \\ 0.40 & -0.36 & -0.08 & 0.13 \end{bmatrix}$	$\begin{bmatrix} 105.8 \\ -99.9 \\ -10.2 \\ 20.4 \end{bmatrix}$
92	$10^6\begin{bmatrix} 3.83 & -3.19 & -1.72 & 2.35 \\ -3.19 & 2.67 & 1.43 & -1.96 \\ -1.72 & 1.43 & 0.92 & -1.21 \\ 2.35 & -1.96 & -1.21 & 1.60 \end{bmatrix}$	$\begin{bmatrix} -22.2 \\ 18.4 \\ 4.8 \\ -7.6 \end{bmatrix}$
93	overflow occurs	

From Table 3.2, it is seen that the covariance matrix had increased from the order of 10^6 at k=1 to 10^{10} at k=50, as there was little or no information about the system dynamics during long periods of steady-state operation. When a unit step change was made in setpoint at k=50 the covariance matrix decreased slowly and reached an order of 10^5 at k=65. However, once this major excitation was over, the magnitude of the covariance matrix grew again with time, as depicted from k=65 to k=92 in Table 3.2.

Theoretically this can be explained as follows. The negative term on the right hand side of equation (3.10) corresponds to the amount of reduction in parameter uncertainty from the last measurement. When the major excitation is over, i.e. setpoint change or load disturbance, there will be no changes in the parameter estimates and the term $\mathbf{P}(k-1)\ \psi(k-1)$ in equation (3.5) will be zero. Consequently $\kappa(k)$ will be zero and so will be the negative term on the right hand side of equation (3.10). At this point, equation (3.10) is practically represented by:

$$\mathbf{P}(k) = \frac{1}{\mu}\ \mathbf{P}(k-1) \tag{3.11}$$

The covariance matrix $\mathbf{P}(k)$ will therefore grow exponentially if μ is less than 1. A large covariance matrix may also cause numerical problems . The numerical problem encountered in this simulation was mainly due to the accumulated effects of roundoff error. At iteration 93, the term $\psi'(k-1)\ \mathbf{P}(k-1)\ \psi(k-1)$ was rounded to -1 such that the gain determined by equation (3.5) was indefinite. Besides the difficulty of choosing a strategy to determine the forgetting factor, the accumulation of roundoff error is the most important disadvantage of using a forgetting factor scheme, and may also lead to negative eigenvalues in the covariance matrix [Ydstie and Sargent, 1984]. The former problem will be solved in the following while the latter will be further

discussed in the next section.

There are many ways suggested by different authors to eliminate the exponential growth of covariance matrix. For instance, the use of an additional perturbation signal, e.g. Pseudo Random Binary Sequence (PRBS), to ensure that the process is properly excited. Other possibilities are to stop the covariance updating when the prediction error is within a given bound, or to use an upper bound on the diagonal elements of the covariance matrix or their trace. Though these techniques have shown some success, there are practical difficulties like determining the *a priori* upper bounds on the diagonal elements. If a measure for the information content in the estimator can be defined and related to the forgetting factor, it seems possible then to determine a forgetting factor such that the covariance matrix stays bounded. This leads to the idea of using a variable forgetting factor [Fortescue, Kershenbaum and Ydstie, 1981; Wellstead and Sanoff, 1981; Isermann, 1982]. This work uses the scheme proposed by Fortescue, Kershenbaum and Ydstie [1981]. Incorporating this into the RLS algorithm described in the previous sections, the term μ in equation (3.10) becomes a variable and is given by:

$$\mu(k) = 1 - \frac{[1 - \psi^t(k-1)\,\kappa(k)]\hat{e}^2(k)}{\Sigma_0} \tag{3.12}$$

$$\text{where } \hat{e}(k) = y(k) - \hat{y}(k) \tag{3.13}$$

$$\Sigma_0 = \sigma^2 N_0 \tag{3.14}$$

N_0 corresponds to the nominal asymptotic memory length and controls the speed of adaptation. A small value of N_0 corresponds to a large covariance matrix and a sensitive system; a large value will result in a less sensitive system but slower adaptation. $\hat{e}(k)$ is the *a posteriori* estimation error. A small error can be interpreted as either: the process has not been excited, e.g. the first 50 iterations in example 3.2; there has been an excitation and the parameters have converged to a set of values; or the estimator has significantly reduced the estimation error. In all these cases, a small *a posteriori* estimation error will result in a forgetting factor value close to 1 (equation 3.12). On the contrary, a smaller forgetting factor will be obtained when the *a posteriori* estimation error is large. Convergence of RLS algorithm with a variable forgetting factor can be found in the work by Osorio Cordero and Mayne [1981] for a deterministic case.

From the viewpoint of programming and computational time, direct implementation of equation (3.12) is not very attractive. However, this implementation can be simplified considerably by replacing $\kappa(k)$ with equation (3.5). The identification algorithm can then be written as:

Predicted output: $\hat{y}(k) = \psi^t(k-1)\ \theta(k-1)$ (3.15)

Estimation error: $\hat{e}(k) = y(k) - \hat{y}(k)$ (3.16)

Gain calculation: $$\kappa(k) = \frac{\mathbf{P}(k-1)\ \psi(k-1)}{[1 + \psi^t(k-1)\mathbf{P}(k-1)\psi(k-1)]} \qquad (3.17)$$

Parameter estimation: $$\hat{\theta}(k) = \theta(k-1) + \kappa(k)\ \hat{e}(k) \qquad (3.18)$$

Forgetting factor:
$$\mu(k) = 1 - \frac{\hat{e}^2(k)}{[1 + \psi^t(k-1)\mathbf{P}(k-1)\psi(k-1)]\Sigma_0} \qquad (3.19)$$

Covariance update:
$$\mathbf{P}(k) = \frac{1}{\mu(k)}[I - \kappa(k-1)\ \psi^t(k-1)]\mathbf{P}(k-1) \qquad (3.20)$$

Though equation (3.19) seems to be more complicated than equation (3.12), a closer look reveals that the term $[1 + \psi^t(k-1)\ \mathbf{P}(k-1)\ \psi(k-1)]$ is first calculated in equation (3.17) and can be stored in the computer for use in equation (3.19). Programming and computational time wise, the variable forgetting factor described by equation (3.19) can now be implemented with one line of FORTRAN code.

The implementation of the above estimation algorithm with its order of execution as written needs a lower bound for $\mu(k)$ to prevent its value from becoming too small or negative. This limit does not need to be specified if a mathematically more involved algorithm is used. In such a case, $\mu(k)$ is solved from a quadratic relationship and it is done before the updating of the gain and parameter estimates. The practical difference in performance of the resulting algorithm is indistinguishable from the simple one [Fortescue, Kershenbaum and Ydstie, 1981; Kershenbaum,

1983].

In addition, the implementation of variable forgetting factor requires *a priori* specification of Σ_0. This specification allows the operator to control the speed of estimator adaptation. From the simulation and experimental studies, it was found that the choice of Σ_0 did not appear to be very sensitive. However, too low a value of Σ_0 could lead to unstable control. When Σ_0 was too small, the forgetting factor was found to remain near the minimum limit. If this continued for a long period of time, it could lead to the blow-up of the covariance matrix (equation 3.11). More discussions on this matter are given in Chapter 4 and Chapter 5.

3.4 Upper-Diagonal Factorization Method

Another problem associated with the conventional RLS algorithm presented in the previous section is the possibility of having indefinite, negative eigenvalues in the covariance matrix. This numerical problem is often due to the high dimension of the covariance matrix and/or the accumulated effects of the roundoff error caused by the finite word length of the computer, i.e. a hardware limitation. Numerous examples illustrating the numerical stability problem of the RLS algorithm can be found in the literature [Kaminski, Bryson and Schmidt, 1971; Bierman, 1976 and 1977].

Potter, who applied the recursive covariance filtering to spacecraft navigation, observed that propagating a square root of the covariance matrix eliminated the problem of an indefinite matrix. Potter's algorithm has been shown to have excellent numerical characteristics and was used successfully in several applications. Despite its numerical stability, Potter's algorithm has found limited usage for several reasons:

1. It takes more computational time and storage than the RLS algorithm.
2. It can only handle scalar measurements.

Andrews [1968] suggested the use of data whitening such that vector measurements can be implemented componentwise. He also suggested the use of triangular matrices to reduce the amount of computational time and storage. However, it was shown later that Andrew's algorithm does not preserve triangularity of the square-root covariance matrix and is equivalent to Potter's algorithm in the scalar case [Kaminski, Bryson and Schmidt, 1973].

The use of triangular square root matrices was further pursued by Carlson [1973]. Bierman [1976], on the other hand, approached the problem by using upper-diagonal factorization method. Even though both methods are algorithmically similar, U-D factorization is more efficient and avoids scalar square roots. Moreover, U-D factorization can be shown to be equivalent to Gentlemen's least-squares algorithm [Gentlemen, 1973], which is based upon the

numerically stable and accurate Givens orthogonal transformation [Bierman, 1977]. One drawback of U-D algorithm is that the updating formulae are not as compact as those of the original RLS algorithm. It is given in a FORTRAN like form. To illustrate this, equation (3.15) to equation (3.20) are rewritten as follows:

```
      Do 1 j = 1, nt
1     ŷ = ŷ + ψ(j)*θ(j)

      ê = y - ŷ

      Do 2 j = nt, 2, -1
      Do 3 k = 1, j-1
3     ψ(j) = ψ(j) + P(k,j)*ψ(k)
2     b(j) = P(j,j)*ψ(j)

      b(1) = P(1,1)*ψ(1)
      a = δ + b(1)*ψ(1)
      γ = 1/a
      P(1,1) = P(1,1)*γ*δ

      Do 4 j = 2, nt
      β = a
      a = a + b(j)*ψ(j)
      h = - ψ(j)*γ
      γ = 1/a
      P(j,j) = P(j,j)*β*γ
      Do 4 k = 1, j-1
      β = P(k,j)
      P(k,j) = β + b(k)*h
4     b(k) = b(k) + b(j)*β

      μ = 1 - ((δ*ê²)/(a*Σ₀))
      IF (μ<μmin) μ = μmin

      Do 5 j = 1, nt
      Do 5 k = 1, nt
5     P(j,k) = P(j,k)/μ

      ê = ê/a

      Do 6 j = 1, nt
6     θ(j) = θ(j) + b(j)*ê
```

The first Do-loop is equivalent to equation (3.15) and followed by the determination of the *a posteriori* estimation error. Do-loops 2, 3 and 4 update the covariance matrix and the gain. It should be noted that the gain $\kappa(k)$ in equation (3.20) has been substituted by $\kappa(k)$ of equation (3.17) in the algorithm. Also, update of the gain $\kappa(k)$ in equation (3.17) has been separated into two parts: the term $\delta + \psi^t(k-1)\ \mathbf{P}(k-1)\ \psi(k-1)$, which in SISO systems is only a scalar, is updated as a and the term $\mathbf{P}(k-1)\ \psi(k-1)$ is stored under b. The original RLS algorithm uses δ as 1, some authors suggest the use of δ as the noise variance of the system and others prefer to replace it with the forgetting factor. In this work, the implementation in the simulation studies has used the noise variance as δ and in the experimental results the value of $\delta=1$ was used. Neither truncation nor indefinite covariance matrix problems were encountered in either case. After the covariance matrix is updated by the factorization method, it is again inflated in Do-loop 5 by the variable forgetting factor. This is important in order to control the speed of adaptation of the estimator. Finally, the parameter estimates are updated in Do-loop 6. Instead of adding a Do-loop to calculate the gain $\kappa(k)$ with b and a, the *a posteriori* estimation error is first divided by a before Do-loop 6. This step is taken to preserving the U-D efficiency. The algorithm can also be modified slightly to store the covariance matrix as a vector such that storage can be reduced in large dimension

problems.

Implementation of the adaptive PID controller with U-D factorization and variable forgetting factor algorithm is straightforward. It does not require any matrix inversion, matrix square roots or trial and error iterative calculations. It involves only simple arithmetic and calculation of the control law is directly determined from the parameter estimates. Therefore, it is computationally efficient and suitable for a microprocessor based computer. Implementation of the algorithm requires the following steps recursively:

1. Measurement of the process output $y(k)$.
2. Prediction of the process output $\hat{y}(k)$, i.e. Do-loop 1.
3. Calculation of the *a posteriori* estimation error $\hat{e}(k)$.
4. Update of the covariance matrix $\mathbf{P}(k)$ and gain $\kappa(k)$, and determination of the forgetting factor $\mu(k)$, i.e. Do-loop 2 to Do-loop 5.
5. Update of the parameter estimates $\theta(k)$, i.e. Do-loop 6.
6. Calculation of the control law, i.e. equation (2.31).

3.5 Summary

This chapter reviews the difficulties commonly encountered when RLS is used for closed-loop control identification, and presents the use of U-D factorization method to implement RLS with a variable forgetting factor. Discussion on the problems of RLS begins with the decrease of estimator gain with respect to time when the process under control has been at steady-state for long duration of time, i.e. when the process has not been 'persistently excited'. This difficulty is solved by introducing a constant forgetting factor, but it is further shown to be inadequate and could lead to blow-up of the covariance matrix when the value of forgetting factor is less than 1. The use of a variable forgetting factor is then introduced. The final section discusses the numerical problems of RLS. It presents the use of U-D factorization algorithm as a solution and outlines the implementation steps of the algorithm with a variable forgetting factor.

4. Simulation Study

4.1 Introduction

The objective of this chapter is to illustrate the properties of the adaptive PID controller presented in Chapter 2, and highlight the factors which should be considered in the 'actual' implementation of the algorithm. Performance of the adaptive PID controller is evaluated through some simulation studies on a hypothetical system, which has also been considered by other authors [Vogel, 1982; Seborg, Shah and Edgar, 1983]. In continuous-time domain, the model of this 'bench-mark' example is given as:

$$\frac{y(s)}{u(s)} = \frac{e^{-ds}}{(3s + 1)(5s + 1)} \tag{4.1}$$

where d is the system time delay ranging from 1 to 5. If a sampling interval of $T_s=1$ is used, the corresponding discrete-time transfer function of the system with a zero order hold is represented by:

$$\frac{y(k)}{u(k)} = \frac{z^{-d-1}(0.028 + 0.0234z^{-1})}{1 - 1.5352z^{-1} + 0.5866z^{-2}} \tag{4.2}$$

In addition, the load disturbance dynamics were modeled by the following continuous-time transfer function:

$$\frac{y(s)}{v(s)} = \frac{1}{1 + 10s} \tag{4.3}$$

The corresponding discrete-time representation with $T_s=1$ is given by:

$$\frac{y(k)}{v(k)} = \frac{0.0952z^{-1}}{1 - 0.9048z^{-1}} \tag{4.4}$$

The following simulation runs were performed to study the effects of:

1. Constant and known time delay
   - Choice of initial b-parameters $\hat{b}(0)$
   - Choice of Σ_0
   - Choice of initial covariance matrix $\mathbf{P}(0)$
2. Constant but unknown time delay
3. Unknown and varying time delay
4. Disturbances and changing process gain
5. Delay dominated systems

on the closed-loop system performance.

4.2 Constant and Known Time Delay

When system time delay is known and constant, many different techniques are available in the literature to compensate for its effect on the overall closed-loop system performance. The Smith Predictor scheme, for example, is one

of the most widely used dead-time compensators. However, due to its sensitivity to modeling errors, a scheme using an adaptive algorithm to estimate the process model parameters and update the compensator was investigated. One of the methods proposed to handle time delay systems is to include additional numbers of $\hat{b}$ parameters in the process model to be estimated. The number of extra parameters can be either equal to the maximum estimated time delay in terms of sampling periods [Wellstead and Sanoff, 1981], or a possible range of time delay expected in the system, i.e. maximum minus minimum time delay [White, 1976]. The former becomes computationally unattractive when system time delay is large. This matter will be further examined in a later section with respect to unknown and/or varying time delay systems.

The following simulation runs were carried out to demonstrate the capability of the adaptive PID controller in handling systems with constant and known time delays. When the system time delay is said to be known, it also means that the number of extra parameters in polynomial $\hat{B}(z^{-1})$ that need to be estimated are the same as the number of sample periods of true delay, i.e. De=Da. The initial 50 sampling periods of the simulation runs are set at zero steady-state condition before any setpoint and/or load changes are introduced. During those periods, white noise $(0,\sigma^2)$ is added to the system with $\sigma^2=0.005$ and the control variable is constrained to $|u(k)|\le 1.0$. After the first 50

sampling periods, the constraint on the control variable is changed to $|u(k)| \leq 100$ and a series of step changes are made in the setpoint to test the controller performance in tracking the desired trajectory.

Figure 4.1 illustrates the simulated servo response of the system when the time delay is one sampling period, i.e. Da=1. Figure 4.2 and Figure 4.9 are the responses when the time delay is two and five sampling periods respectively. Though the control variable is clipped during the initial 50 sampling periods, simulation runs show that the control variable is well within the limit during those periods. The output variable is shown to follow the desired trajectory very closely in all cases. As the time delay increases, the amount of overshoot on the output variable increases. This is because of the longer period of uncertainty in the parameter estimates during the initial period. However, this overshoot decreases as time increases (Figure 4.9).

Figure 4.3a to Figure 4.3c show the convergence of the estimated parameters when time delay is two sampling periods. Derivation of the adaptive PID controller in Chapter 2 indicates that the process model be a second order. There are therefore four parameters to be estimated in each case in addition to the extra parameters used to compensate for the time delay. For the case in Figure 4.3a to Figure 4.3c, it amounts to six parameters. Each parameter is shown to fluctuate during the initial period. This is expected since each parameter is initialized to zero except

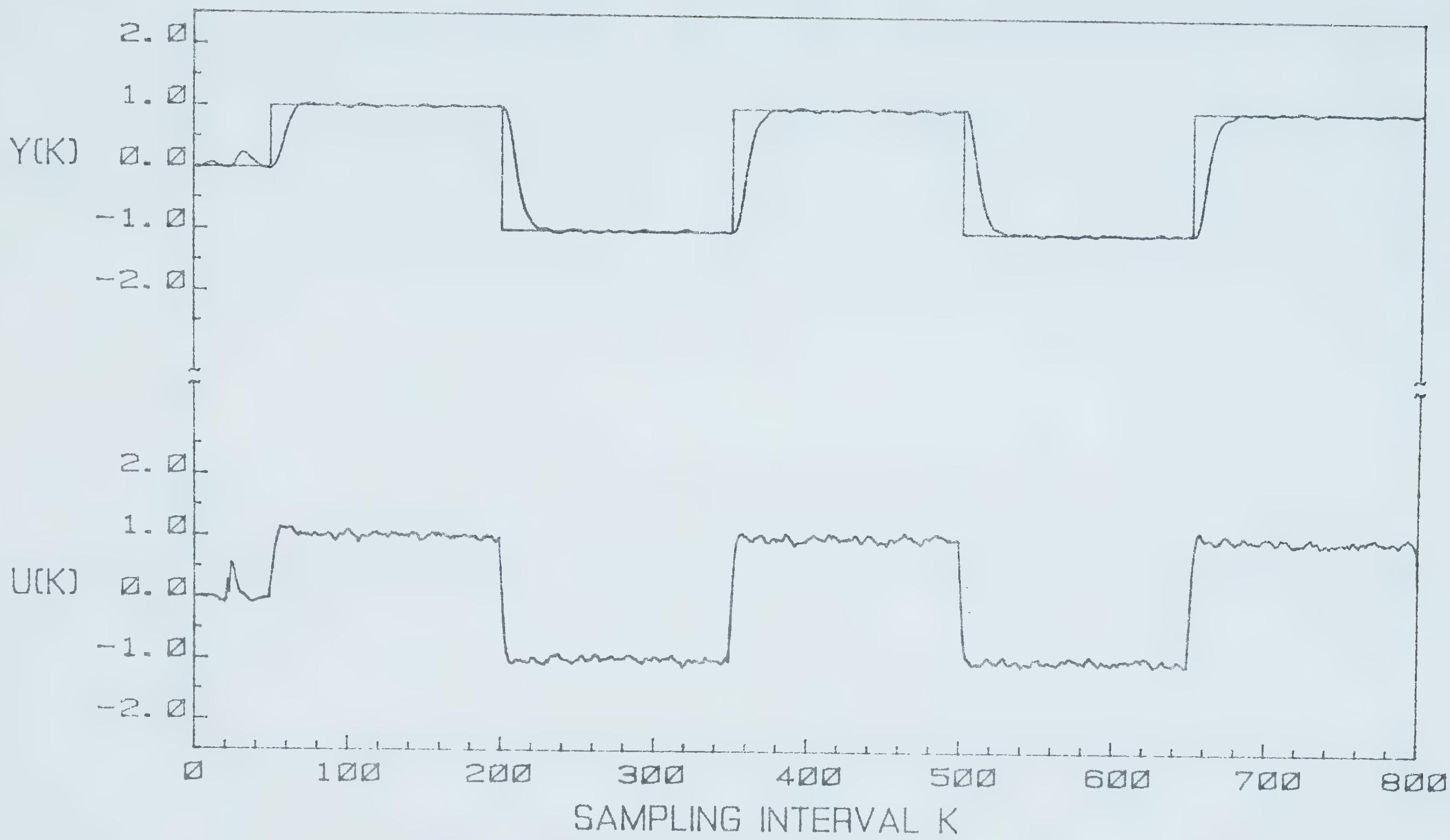

Figure 4.1. Simulated servo response using adaptive PID controller (Da=De=1/M=4/W=0.8/So=1/Po=1.0E+06)

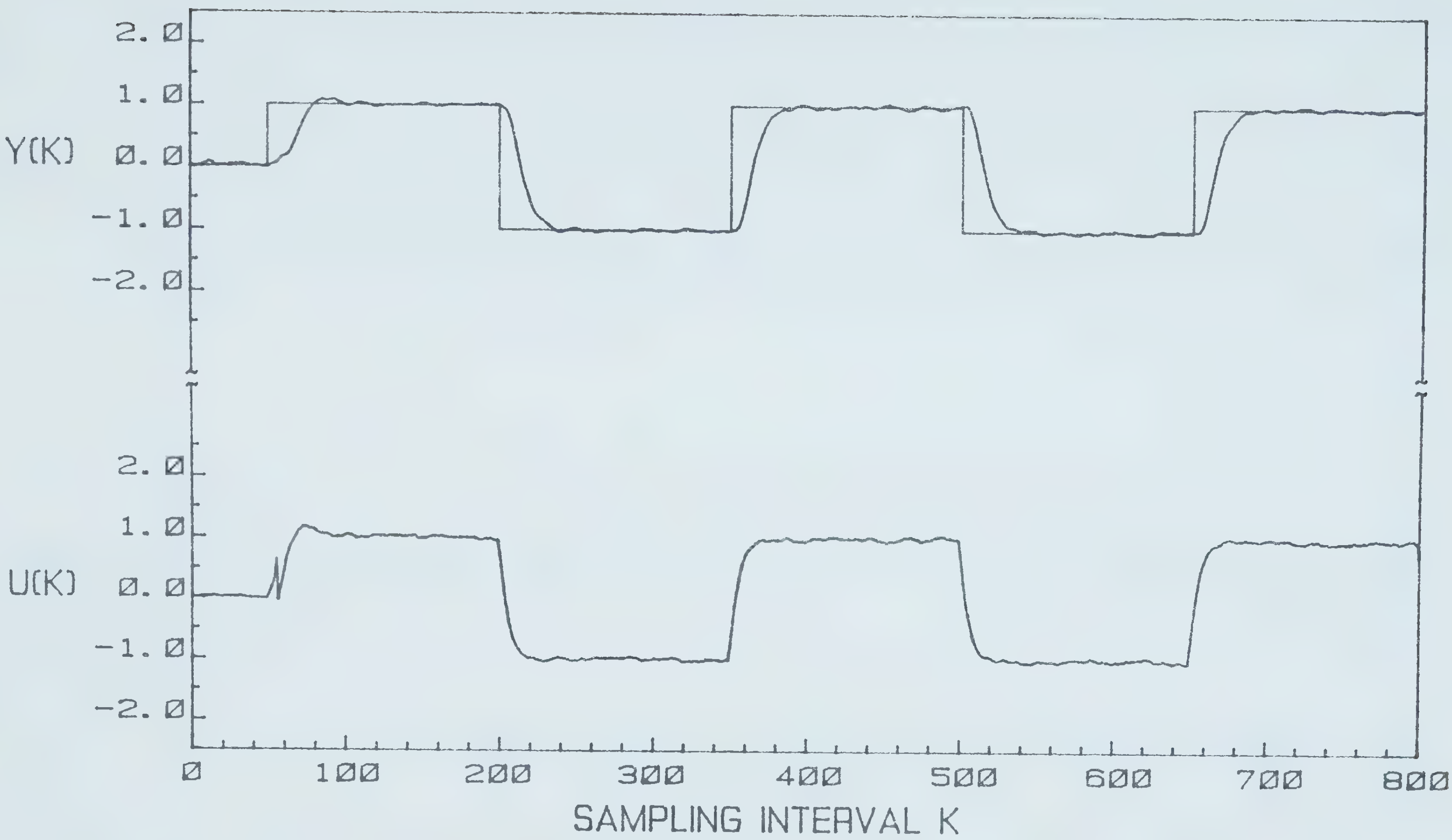

Figure 4.2. Simulated servo response using adaptive PID controller (Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

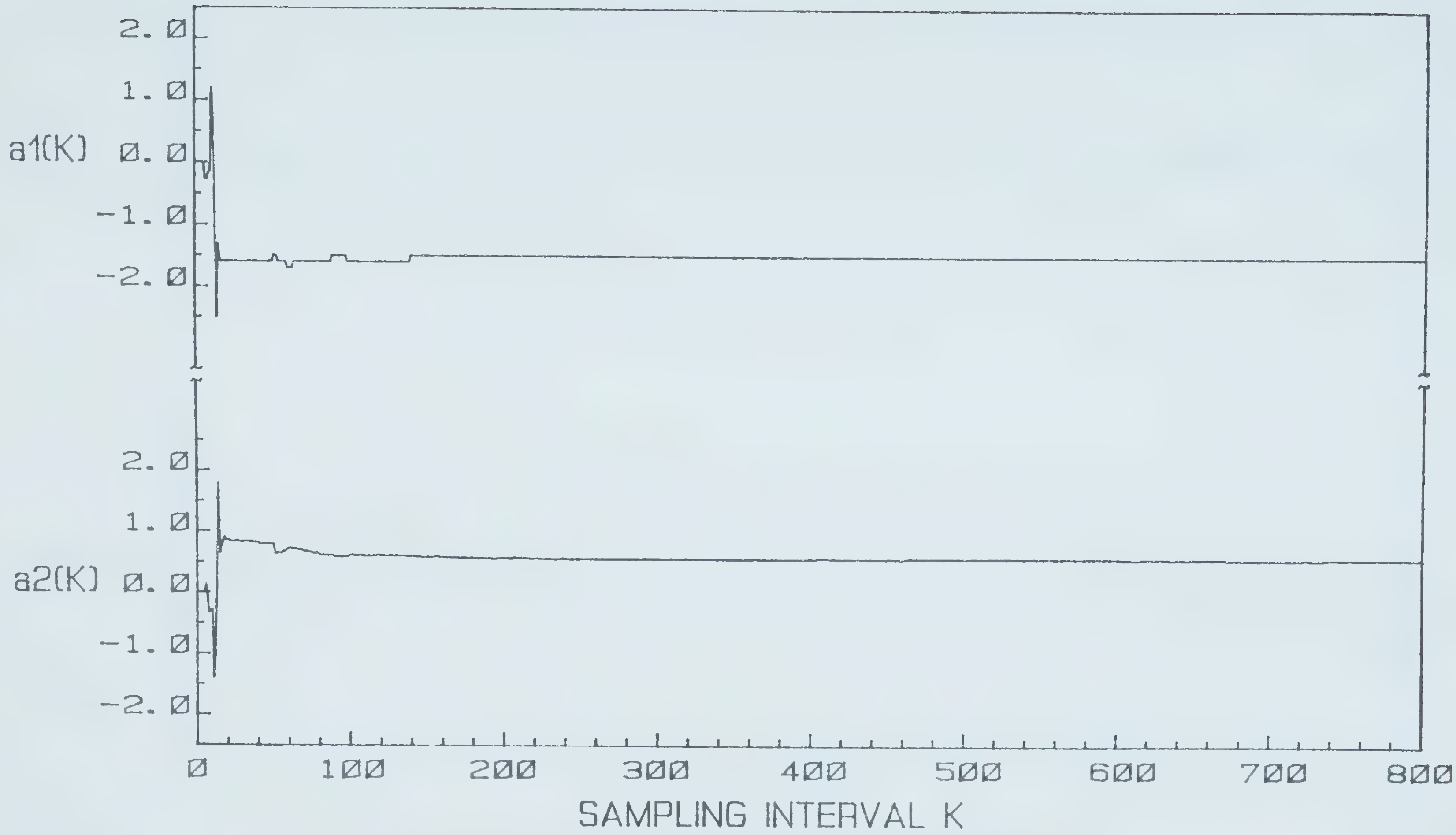

Figure 4.3a. Parameter convergence of adaptive PID controller (Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

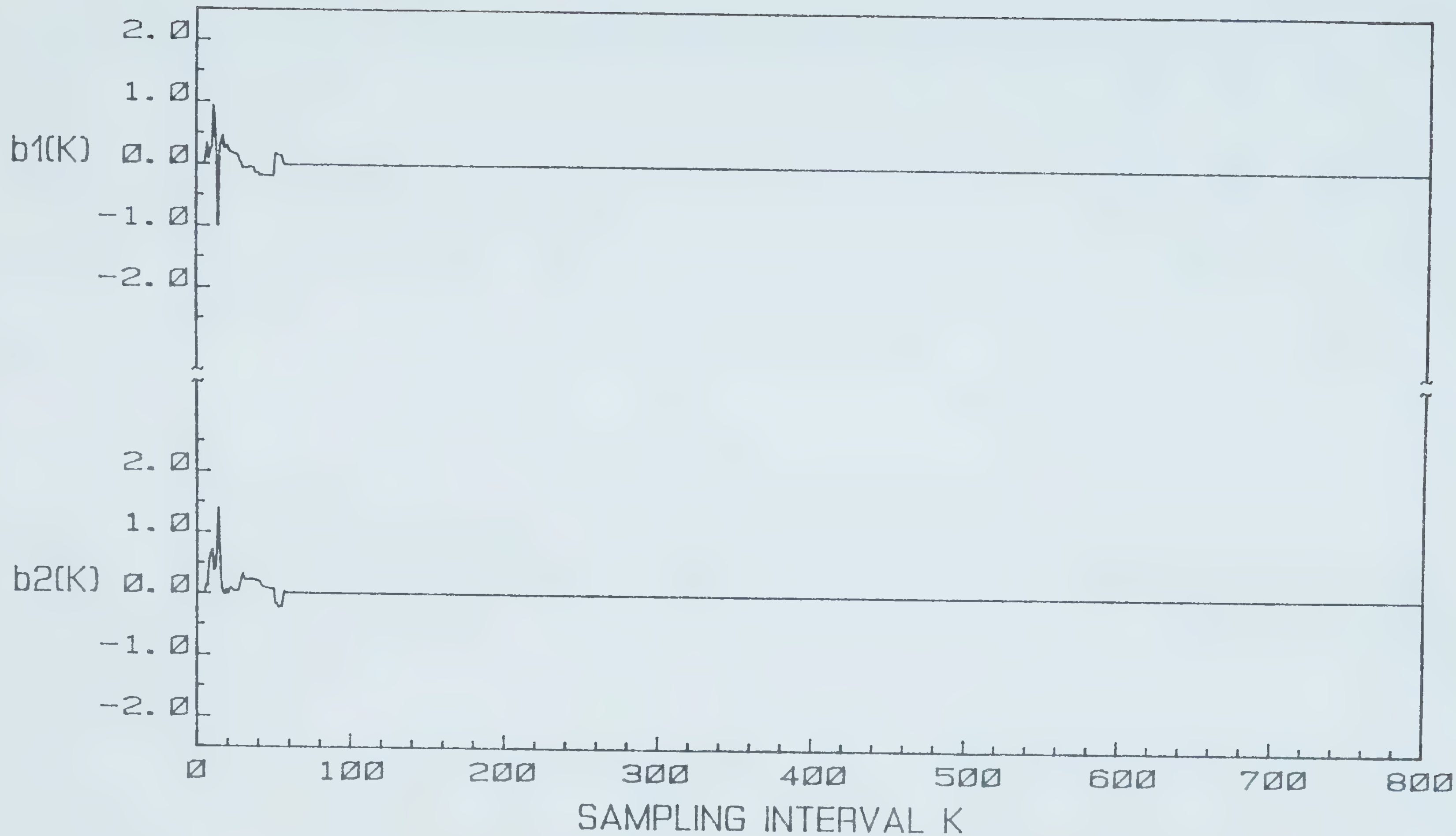

Figure 4.3b. Parameter convergence of adaptive PID controller (Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

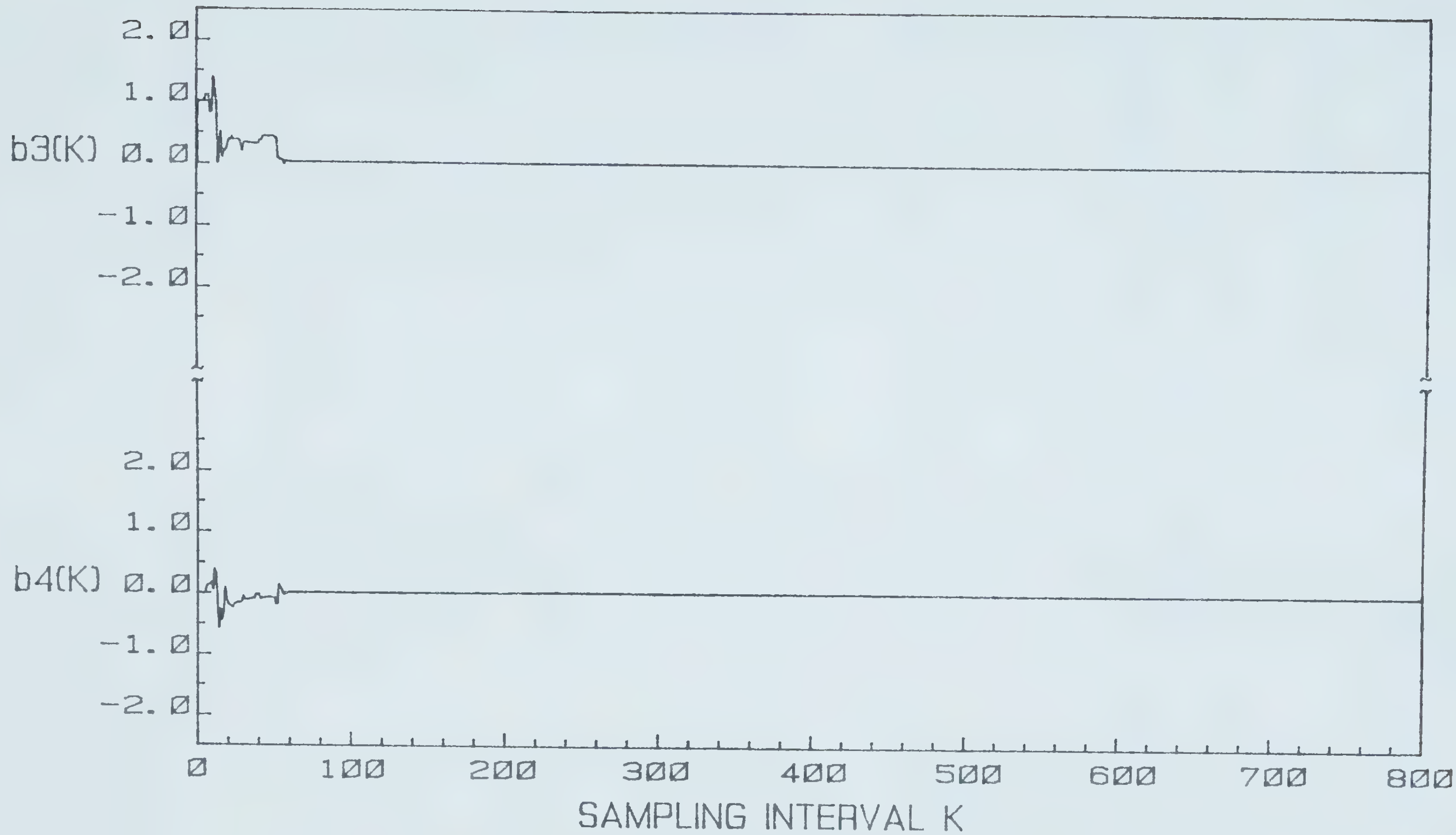

Figure 4.3c. Parameter convergence of adaptive PID controller
(Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

for $\hat{b}_3(0)$. However, all parameters converge rapidly after the introduction of the first step change in the setpoint. Below is a list of the 'true' process parameters and the estimated parameters.

Table 4.1. List of Parameter Estimates When System Time Delay is Known and Constant

Parameter	'True' Parameters	Estimated Parameters
a_1	-1.5352	-1.5
a_2	0.5866	0.57
b_1	0.0	-0.0022
b_2	0.0	0.0049
b_3	0.0280	0.0240
b_4	0.0234	0.0280

The three PID controller settings are shown in Figures 4.4a, 4.4b and 4.4c to behave in a similar fashion. This is due to the fact that they are determined from the estimated parameters according to equations (2.37), (2.38) and (2.39). Figure 4.5 shows the variable forgetting factor. The change in the value of forgetting factor is so small that it appears as if it stayed at a constant value at 1. The choice of the lower limit of forgetting factor is important. This point will be further discussed in connection with the choice of Σ_0. For all the simulation runs, the lower limit is set to 0.9.

The adaptive PID controller algorithm also provides an on-line tuning parameter, i.e. pole-location w_1, to allow the operator to 'shape' the output response. A small value of w_1 corresponds to fast rise time and a large value gives

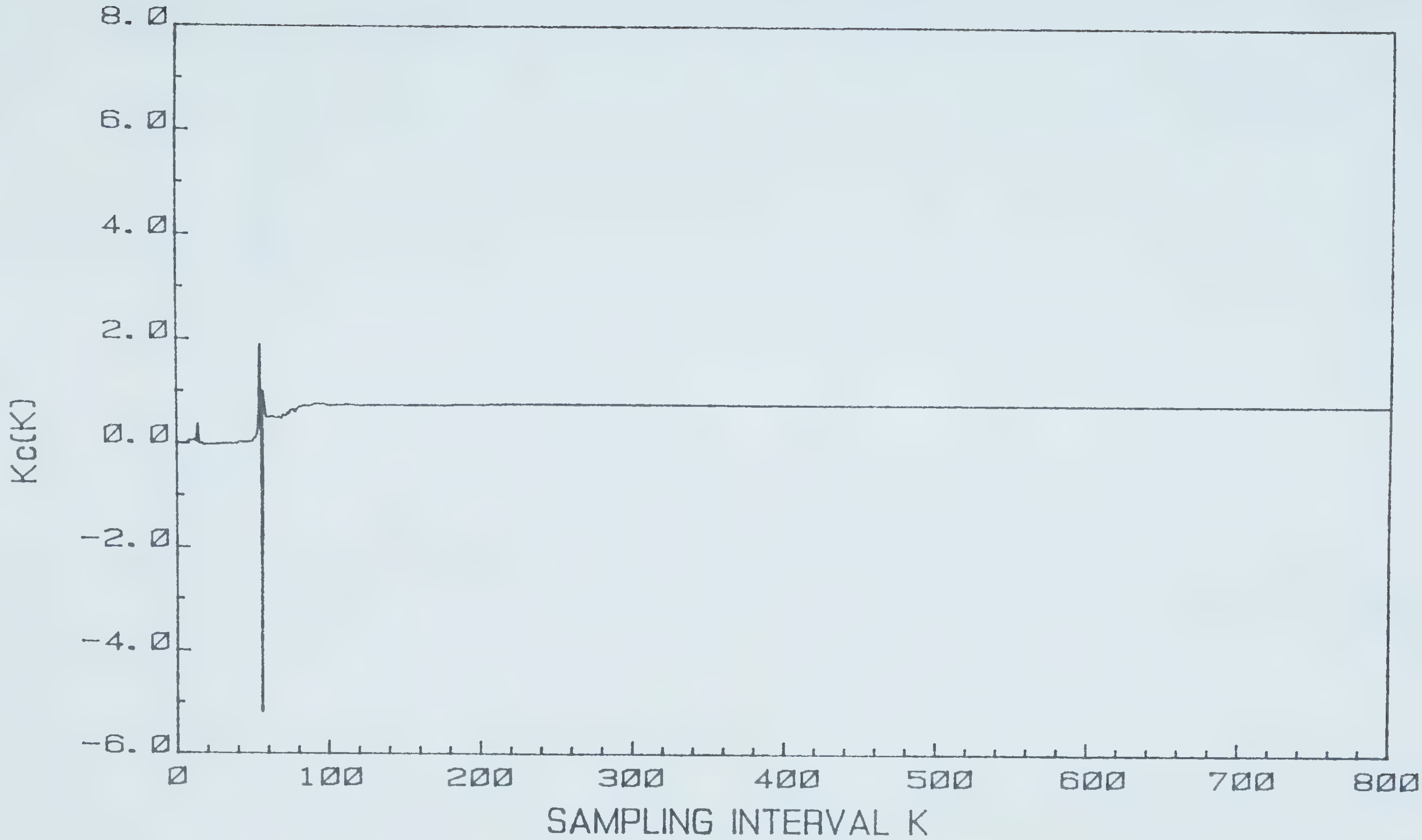

Figure 4.4a. Controller settings of adaptive PID controller
(Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

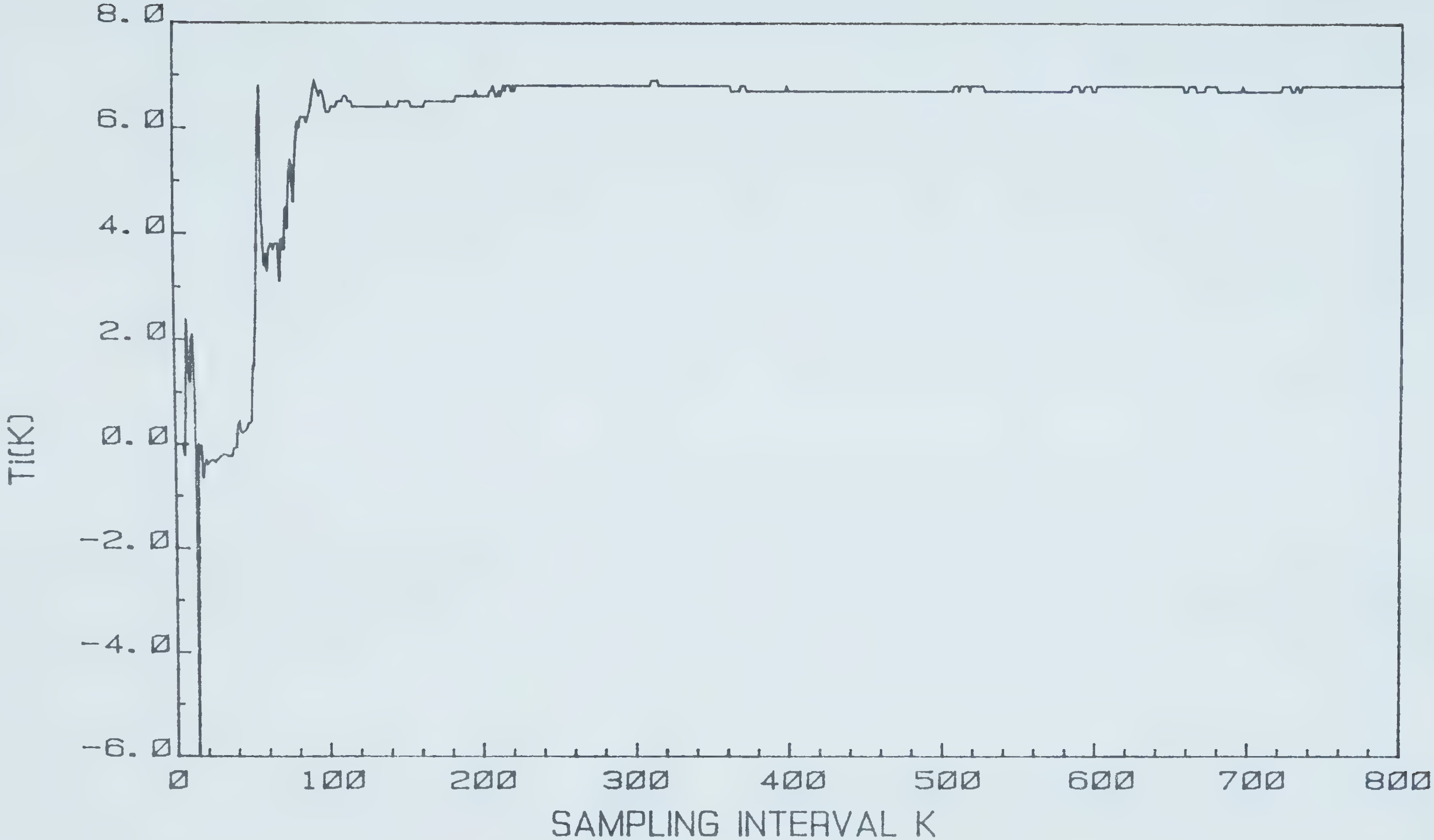

Figure 4.4b. Controller settings of adaptive PID controller

(Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

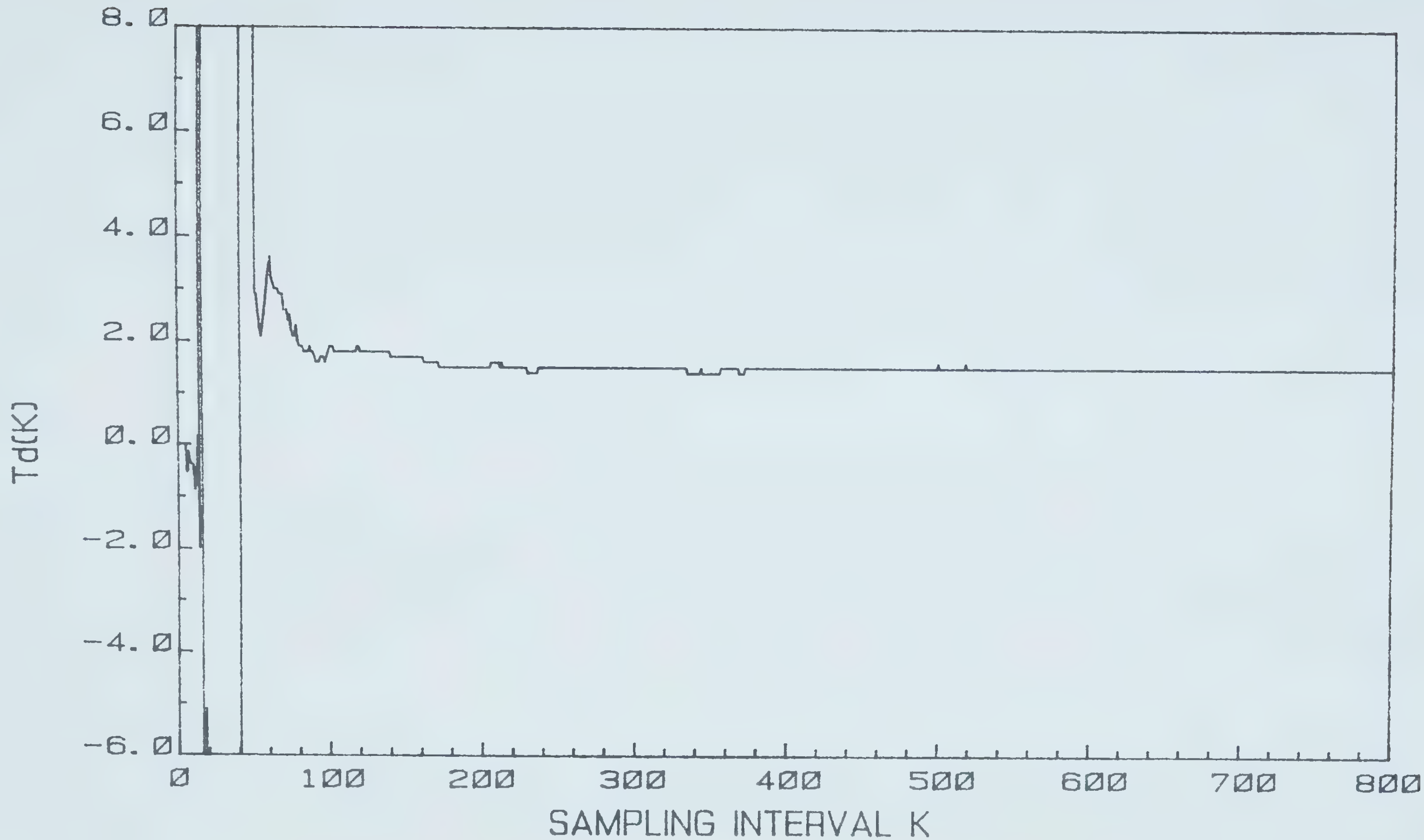

Figure 4.4c. Controller settings of adaptive PID controller

(Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

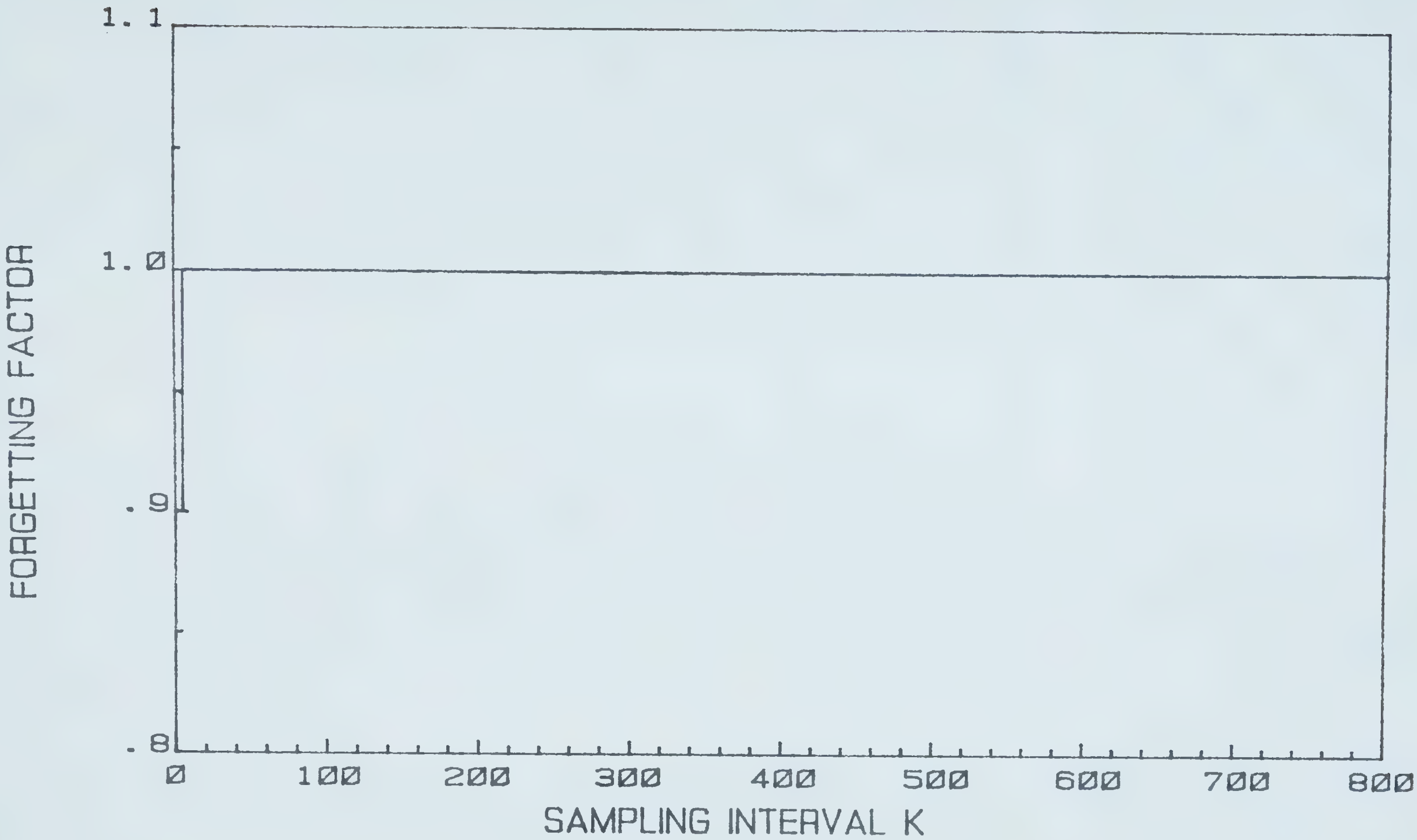

Figure 4.5. Forgetting factor of adaptive PID controller
(Da=De=2/M=5/W=0.9/So=1/Po=1.0E+06)

sluggish response. Since the value of $\Sigma\hat{b}_i$ is used in place of $B(z^{-1})$ in equation (2.24), it follows that the control action is actually faster than it would have been with $B(z^{-1})$. Due to this reason, it was found that small value of w_1 gives very fast rise time and causes the response to become oscillatory. For the simulation studies, a value of 0.9 is used except for Figure 4.1 where w_1=0.8.

Choice of Initial b-Parameters $\Sigma\hat{b}_i$

From the control law described by equations (2.32) and (2.33), it is obvious that initial parameter estimates for the $\hat{b}$'s cannot be all set to zero since this implies that the control input during the initial period will be indefinite. To start the U-D estimation algorithm, all the initial parameters are therefore set to zero except for one parameter in polynomial $\hat{B}(z^{-1})$. Theoretically any one of the $\hat{b}$ parameters can be initialized to non-zero. To see if this is true, two simulation runs with different $\hat{b}$ parameters initialized to non-zero were carried out with the remaining conditions being identical. Both give similar results. Figure 4.2 shows the simulated response when $\hat{b}_3$ is initialized to a non-zero value, and Figure 4.6 shows the case when $\hat{b}_2$ is initialized to a non-zero value.

Choice of Σ_0

The value of Σ_0 given by equation (3.15) is proportional to the nominal asymptotic memory length and the

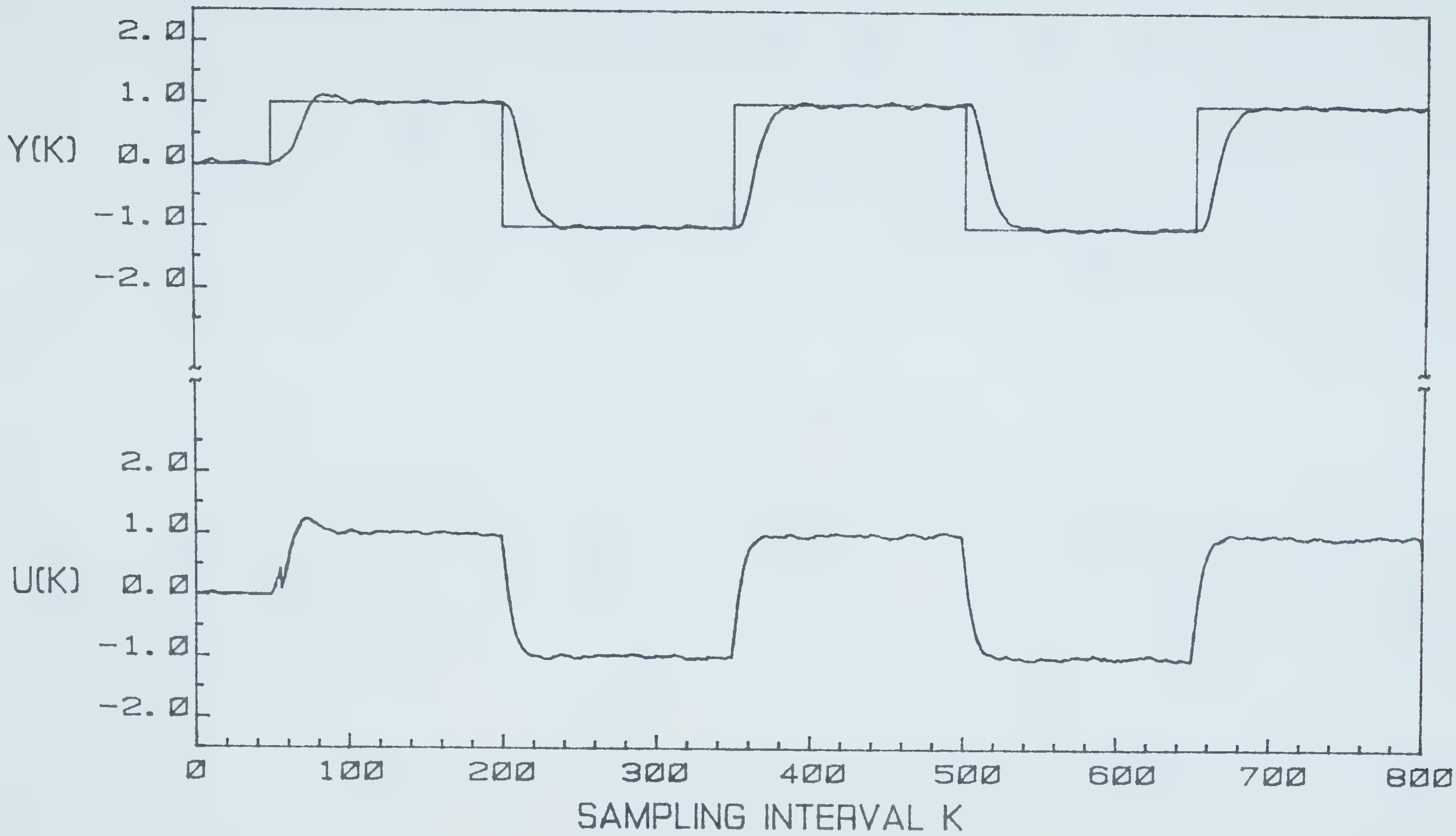

Figure 4.6. Simulated servo response using adaptive PID controller (Da=De=2/M=4/W=0.9/So=1/Po=1.0E+06)

system noise variance. From equation (3.13), a small value of Σ_0 will cause the forgetting factor to decrease and it may even hit the minimum limit. A small forgetting factor corresponds to a large covariance. This implies that the system is sensitive to disturbances and speed of adaptation is faster in response to sudden parameter changes. However, small values of Σ_0 do not necessarily give better control. For noisy systems, small values of Σ_0 will cause the forgetting factor to remain near the minimum limit most of the time and can thus create an oversensitive system. The value of Σ_0 should, in this case, be chosen high enough to include the noise variance as dictated by equation (3.15). On the other hand, this observation also provides an indication as to which direction the value of Σ_0 should be changed to, i.e. to increase or decrease. Figure 4.2 and Figure 4.7 show the results of two different runs with $\Sigma_0=1$ and $\Sigma_0=10$ respectively. Both runs appear to be similar.

Choice of Initial Covariance Matrix P(0)

In general, a large initial covariance matrix indicates that the level of confidence in the initial estimates of the parameters is low and the estimator is thus required to response fast to parameter changes. When *a priori* knowledge about the controlled process is unavailable, as is often the case, the initial covariance matrix is usually set to a relatively large diagonal matrix. Using the U-D estimation algorithm, this choice is found to be insensitive. Figure

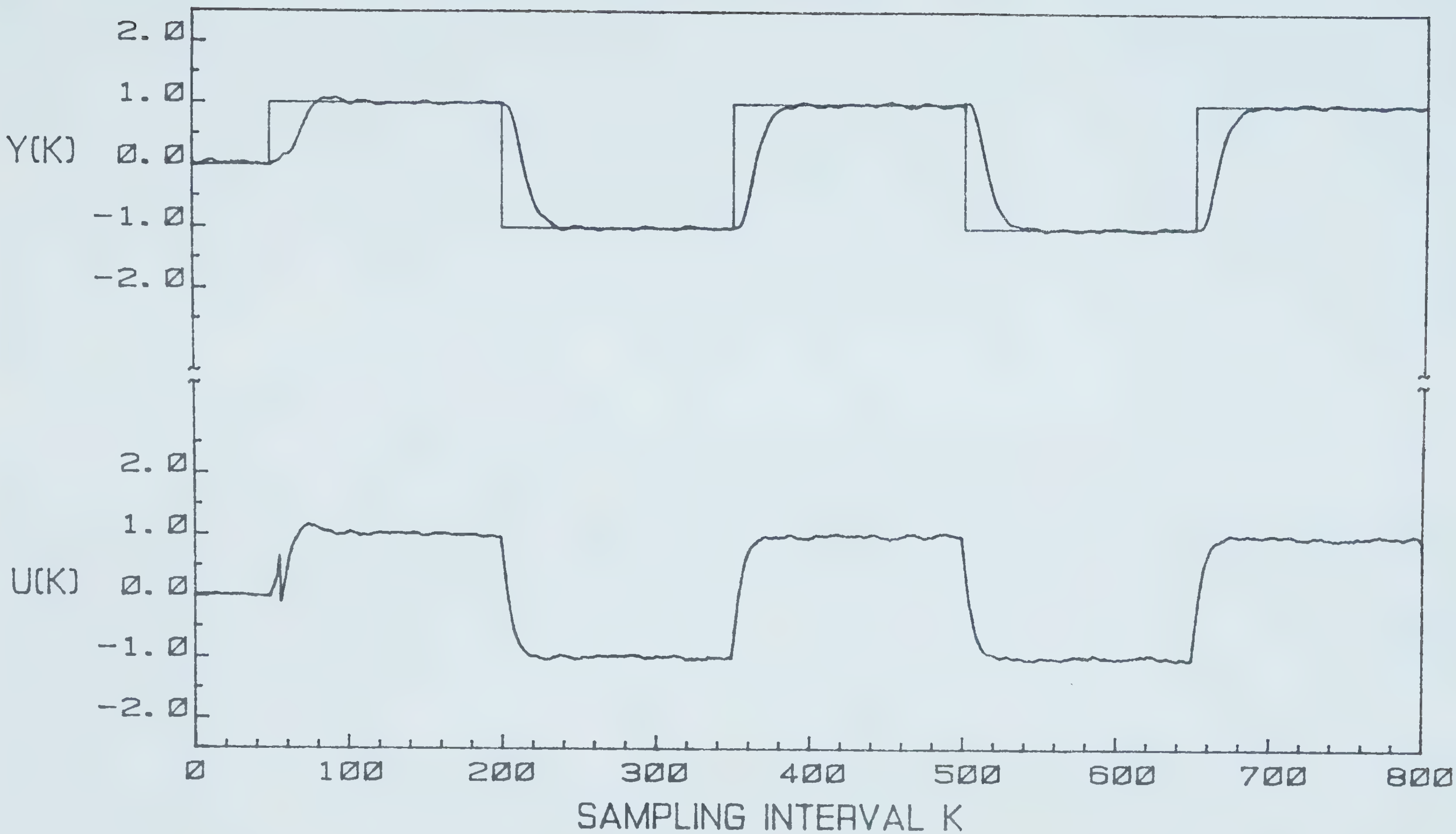

Figure 4.7. Simulated servo response using adaptive PID controller (Da=De=2/M=5/W=0.9/So=10/Po=1.0E+06)

4.2 and Figure 4.8 give the closed-loop system responses with $\mathbf{P}(0)=10^6\mathbf{I}$ and $\mathbf{P}(0)=10^2\mathbf{I}$ respectively. Though Figure 4.8 gives a higher overshoot during the first step change in setpoint, there is practically no difference as time increases. Since the choice of $\mathbf{P}(0)$ was found to be insensitive, with the exception of Figure 4.8, the initial covariance matrix $\mathbf{P}(0)$ for all simulation runs is set to $10^6\mathbf{I}$.

4.3 Constant but Unknown Time Delay

For systems with unknown time delay, it is a common practice to tune the controller by using the maximum value of the expected delay. Consequently, the system response often becomes sluggish. Two simulation runs were performed to test the performance of the adaptive PID controller to a system with unknown time delay by overparameterizing and underparameterizing the polynomial $\hat{B}(z^{-1})$. By overparameterization it is meant that the number of extra parameters is greater than the maximum expected time delay in terms of sampling periods. For example, if the maximum time delay is expected to be 4 sampling periods, instead of identifying 4 extra parameters in $B(z^{-1})$, overparameterization identifies 5 or 6 extra parameters. Similarly, by underparameterization it is meant that the number of extra parameters is less than the maximum expected time delay. Using the above example, the number of extra parameters identified in underparameterization will be

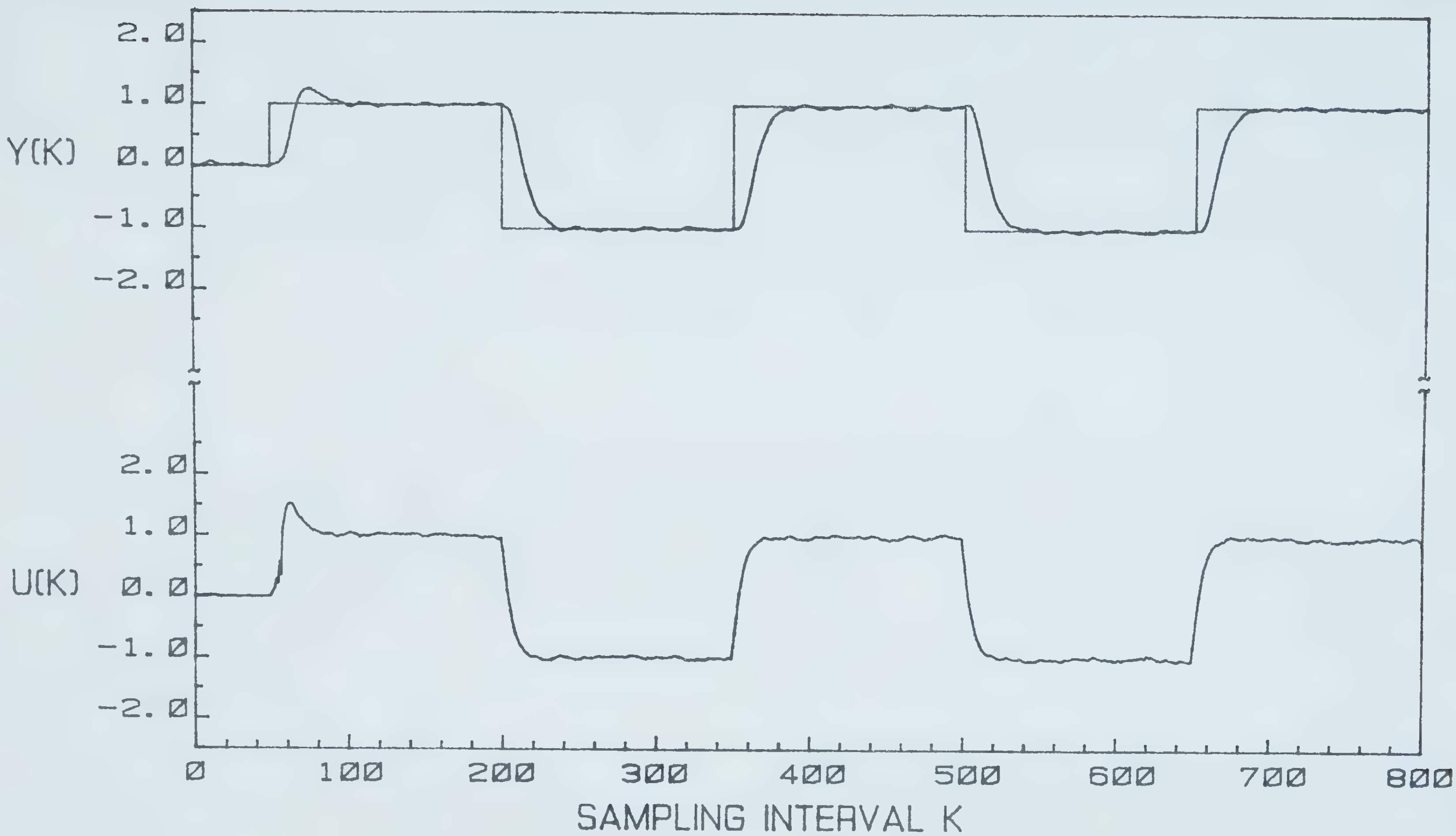

Figure 4.8. Simulated servo response using adaptive PID controller (Da=De=2/M=5/W=0.9/So=1/Po=1.0E+02)

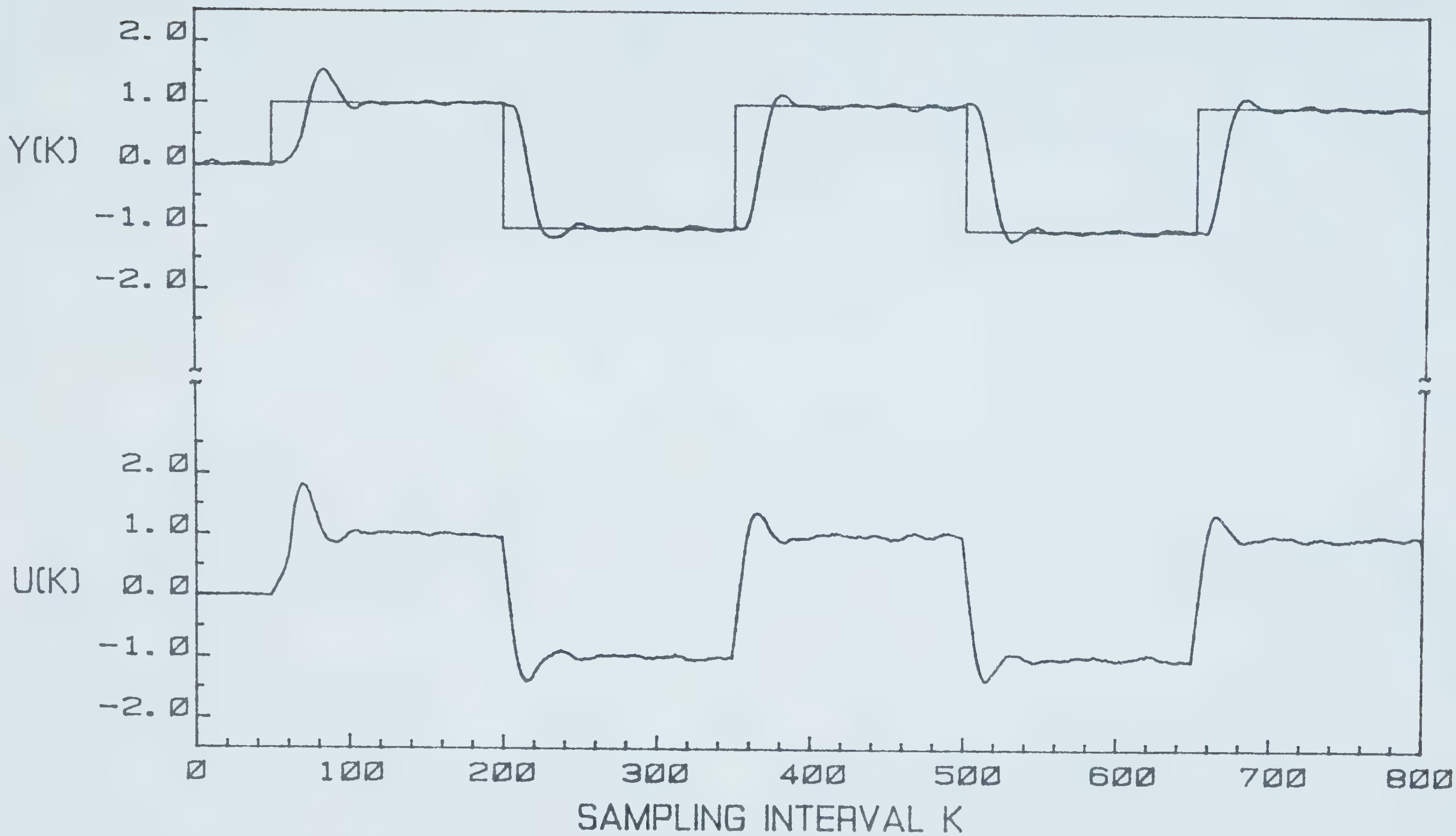

Figure 4.9. Simulated servo response using adaptive PID controller (Da=De=5/M=8/W=0.9/So=1/Po=1.0E+06)

anything from 1 to 3.

In Figure 4.10 the number of extra parameters is greater than the maximum expected time delay, i.e. six extra parameters. Comparison of this run to the one when time delay is known, Figure 4.2, shows similar responses. Figure 4.11 displays the system performance when the number of extra parameters is less than the maximum expected time delay. The run in Figure 4.11 has employed two extra parameters in polynomial $\hat{B}(z^{-1})$. Theoretically, this indicates to the estimator that there are two sampling periods of delay. The actual time delay in the system, however, is five sampling periods. Nevertheless, giving a sequence of input-output history, the estimator will converge to a set of parameters which also minimizes the estimation error. Since the control law described by equations (2.32) and (2.33) uses only the value of $\Sigma\hat{b}_i$ and not the individual $\hat{b}_i$, the ultimate effect is therefore equivalent. If this run is compared to Figure 4.9 with known time delay, it can be seen that the rise time during the first step change is faster with the consequence of a larger overshoot. This is due to a larger uncertainty during the initial period in the parameter estimates. As time increases, Figure 4.11 shows smaller overshoot than Figure 4.9 with no significant difference in rise time. In addition, there is a significant amount of computational time to be gained in underparameterization since less parameters are estimated.

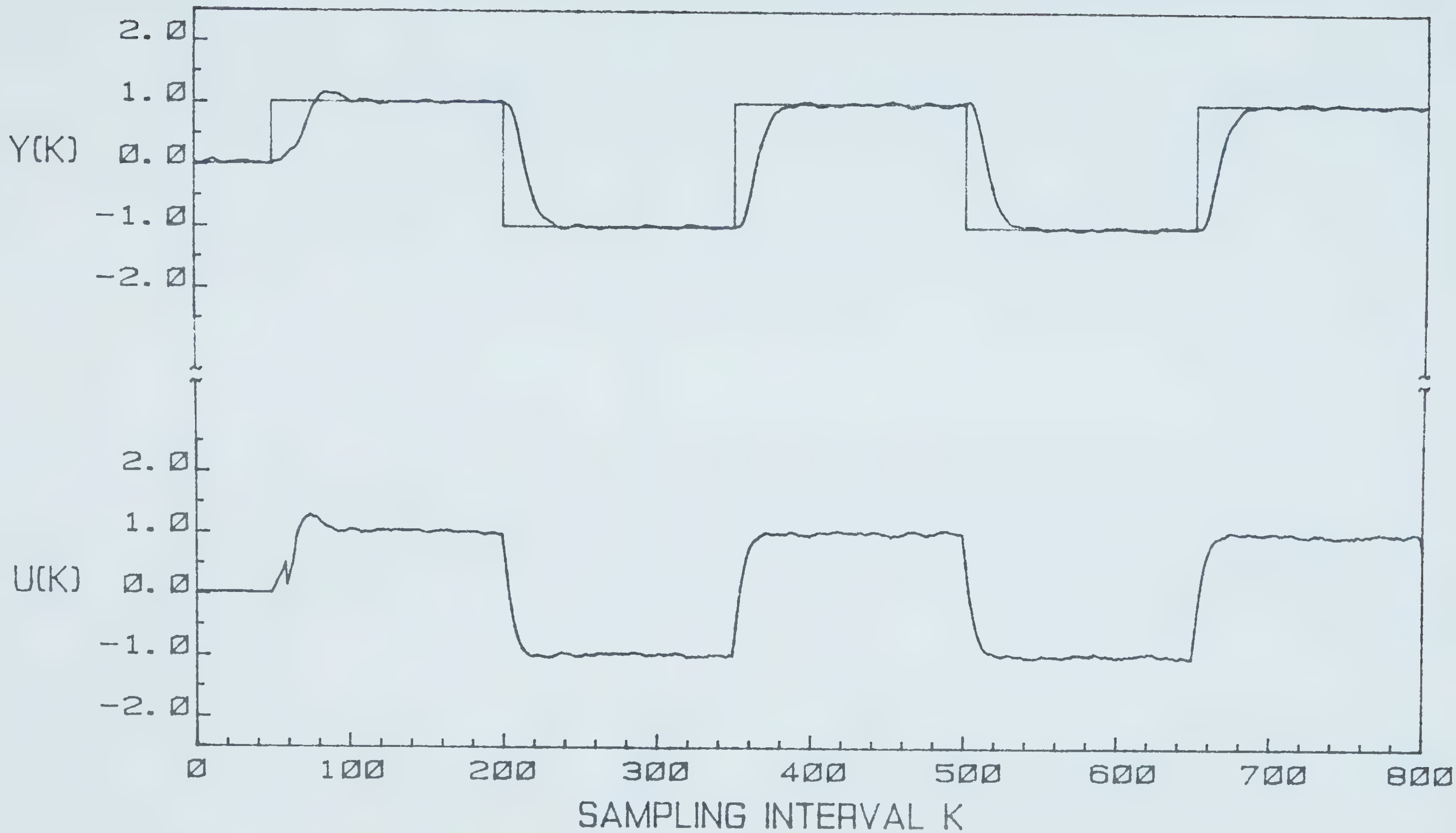

Figure 4.10. Simulated servo response using adaptive PID controller (De=6/Da=2/M=9/W=0.9/So=1/Po=1.0E+06)

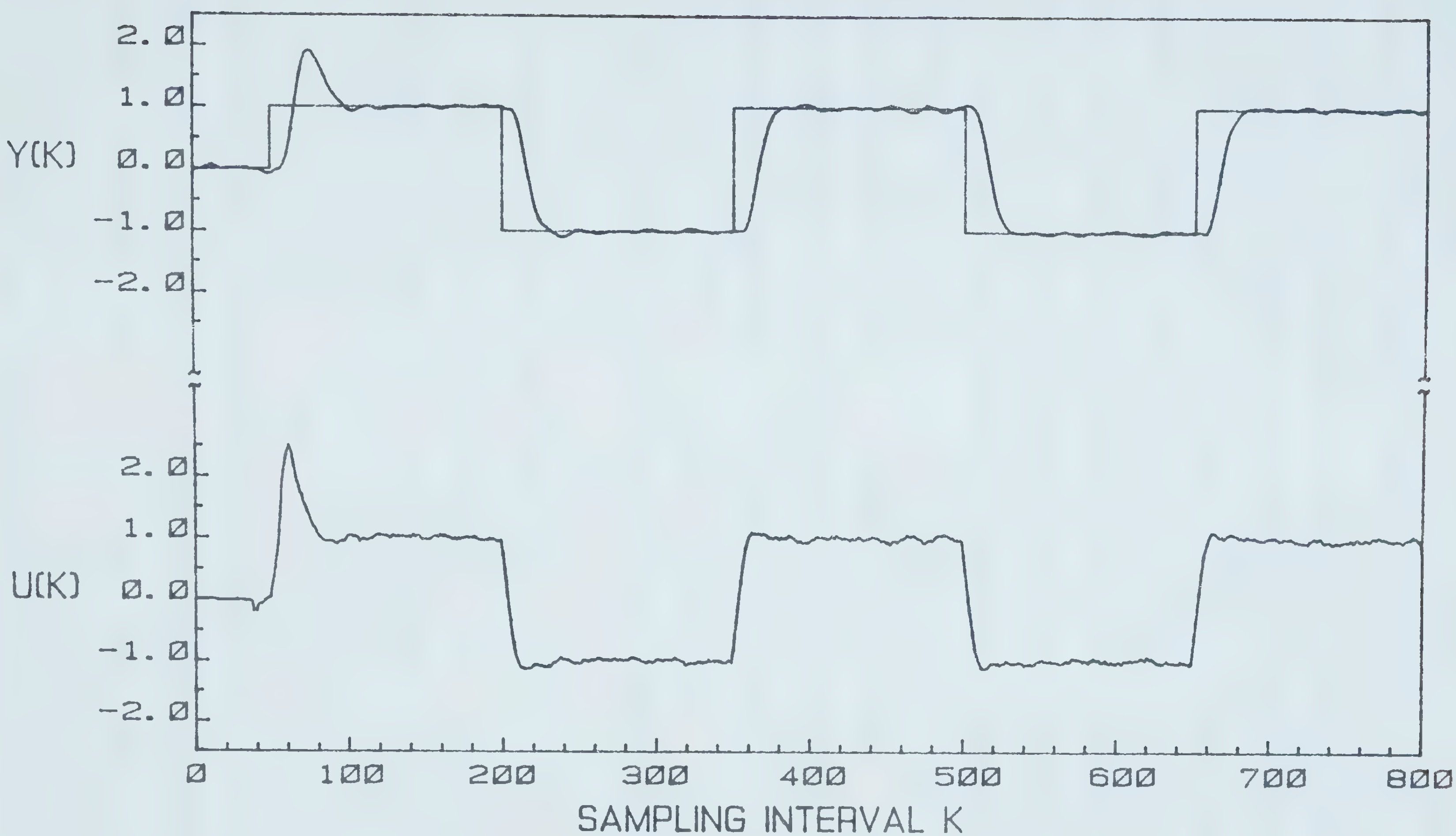

Figure 4.11. Simulated servo response using adaptive PID controller (De=2/Da=5/M=5/W=0.9/So=1/Po=1.0E+06)

Wellstead and Sanoff [1981] proposed to overparameterize the polynomial $\hat{B}(z^{-1})$ by the largest expected time delay. White[1976], on the other hand, recommended the overparameterization of polynomial $\hat{B}(z^{-1})$ by an expected range of time delay to reduce the total number of estimated parameters. When the minimum expected time delay in White's approach is set to zero, the two approaches can be shown to be equivalent. The whole purpose of providing *a priori* estimation of the minimum time delay is to reduce the total number of parameters estimated when the maximum time delay is expected to be large. The underparameterization scheme described above has demonstrated the independence of the adaptive PID controller performance from the total number of extra parameters in polynomial $\hat{B}(z^{-1})$. This algorithm is therefore computationally more efficient due to this fact alone.

4.4 Unknown and Varying Time Delay

Inspite of the difficulties they create in control problems, systems with unknown and/or varying time delay characteristics are often encountered in chemical processes. A typical example involves the transport delays in chemical processes which vary with the process flowrate. From section 4.3, it is shown that the adaptive PID controller can perform equally well even when the number of extra parameters estimated is less than the maximum expected time delay. Similar tests are performed for systems with unknown

and varying time delay and the results are given in this section.

Figure 4.12 and Figure 4.16 show the performances of the adaptive PID controller when the system time delay is unknown and changes. In Figure 4.12, the time delay changes from two to five sampling periods at k=500, and only two extra parameters are included in polynomial $\hat{B}(z^{-1})$. When the time delay is five sampling periods, the number of extra parameters is no longer equal to the actual delay but less. However, system performance is not degraded since the algorithm uses only the steady-state value, $\Sigma\hat{b}_i$, of polynomial $\hat{B}(z^{-1})$.

When the time delay changes, though the process parameters might be invariant, there is a mismatch in the process model that causes the parameters to change to accomodate for the total effect. The parameter convergence is rapid and is shown in Figure 4.13, 4.14a and 4.14b. For comparison purposes, the 'true' process parameters and the estimated parameters are listed in Table 4.2. Along with the changes of parameters at k=500, the estimation error increases. Consequently, the forgetting factor also decreases to allow the estimator to adapt faster to the changes. Once the parameters converge, the value of forgetting factor goes back to one. This is shown in Figure 4.15.

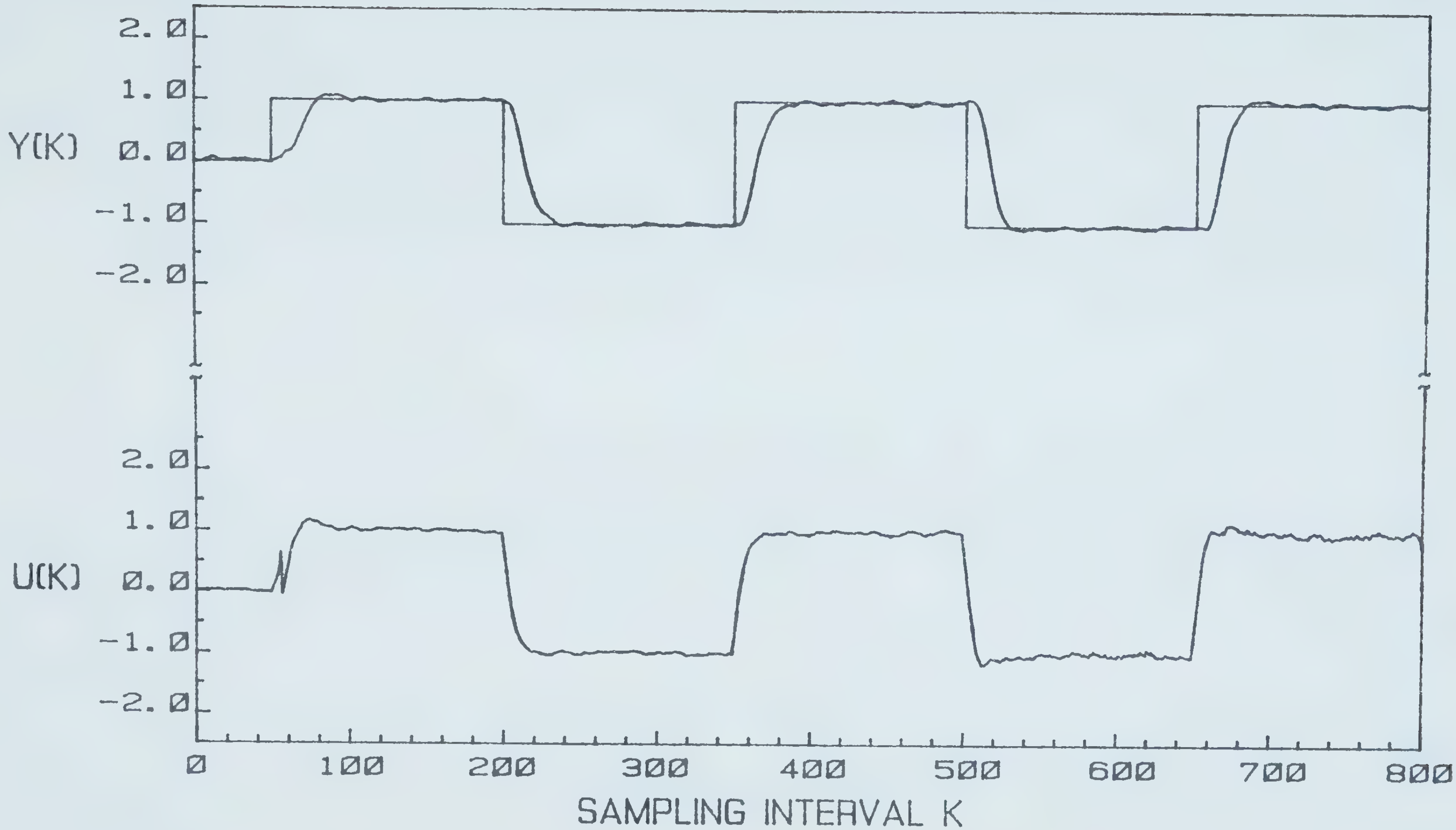

Figure 4.12. Simulated servo response using adaptive PID controller (De=2/Da(0<K<500)=2/Da(K>500)=5/M=5/W=0.9/So=1/Po=1.0E+06)

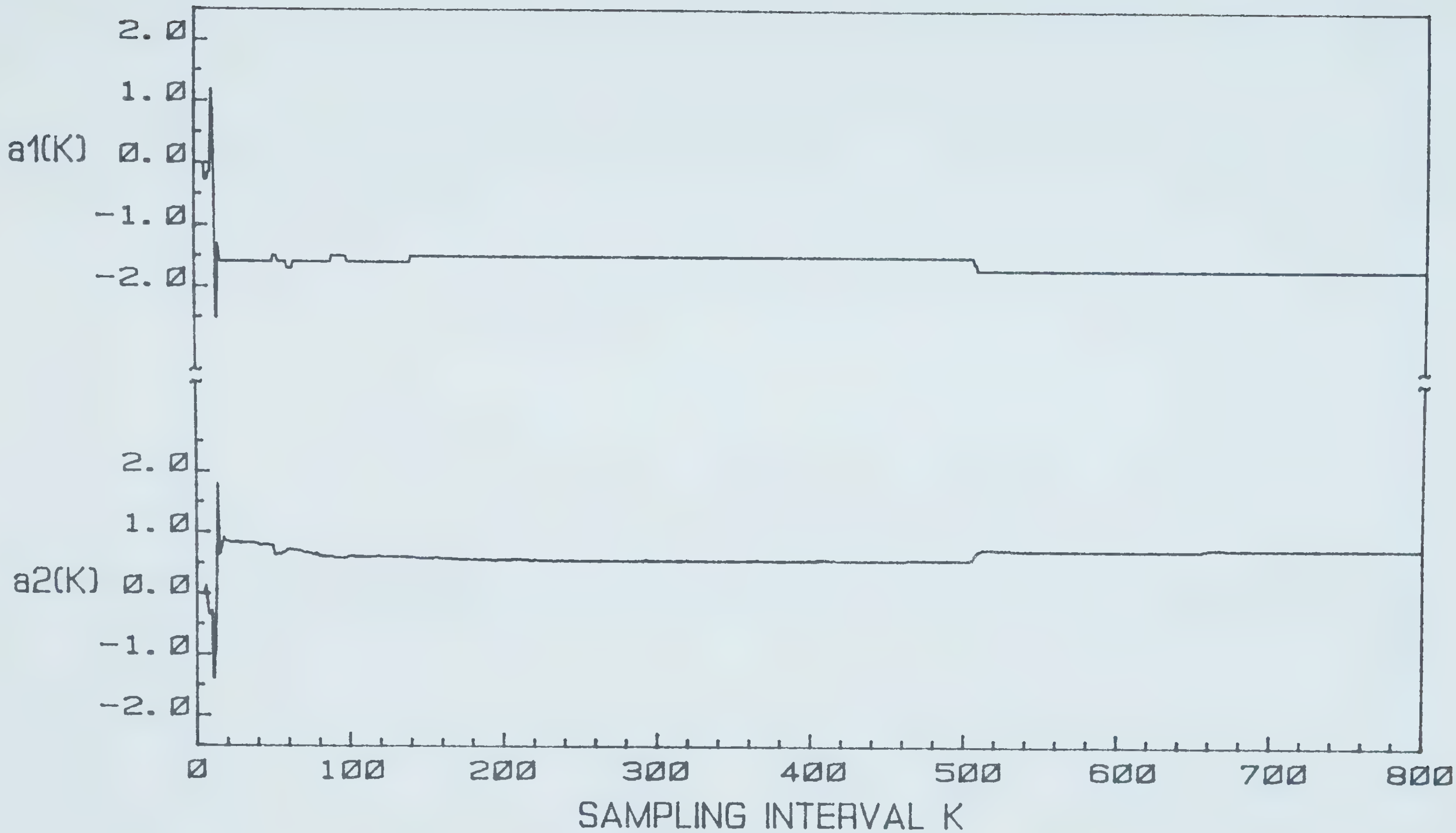

Figure 4.13. Parameter convergence of adaptive PID controller

(De=2/Da(0<K<500)=2/Da(K>500)=5/M=5/W=0.9/So=1/Po=1.0E+06)

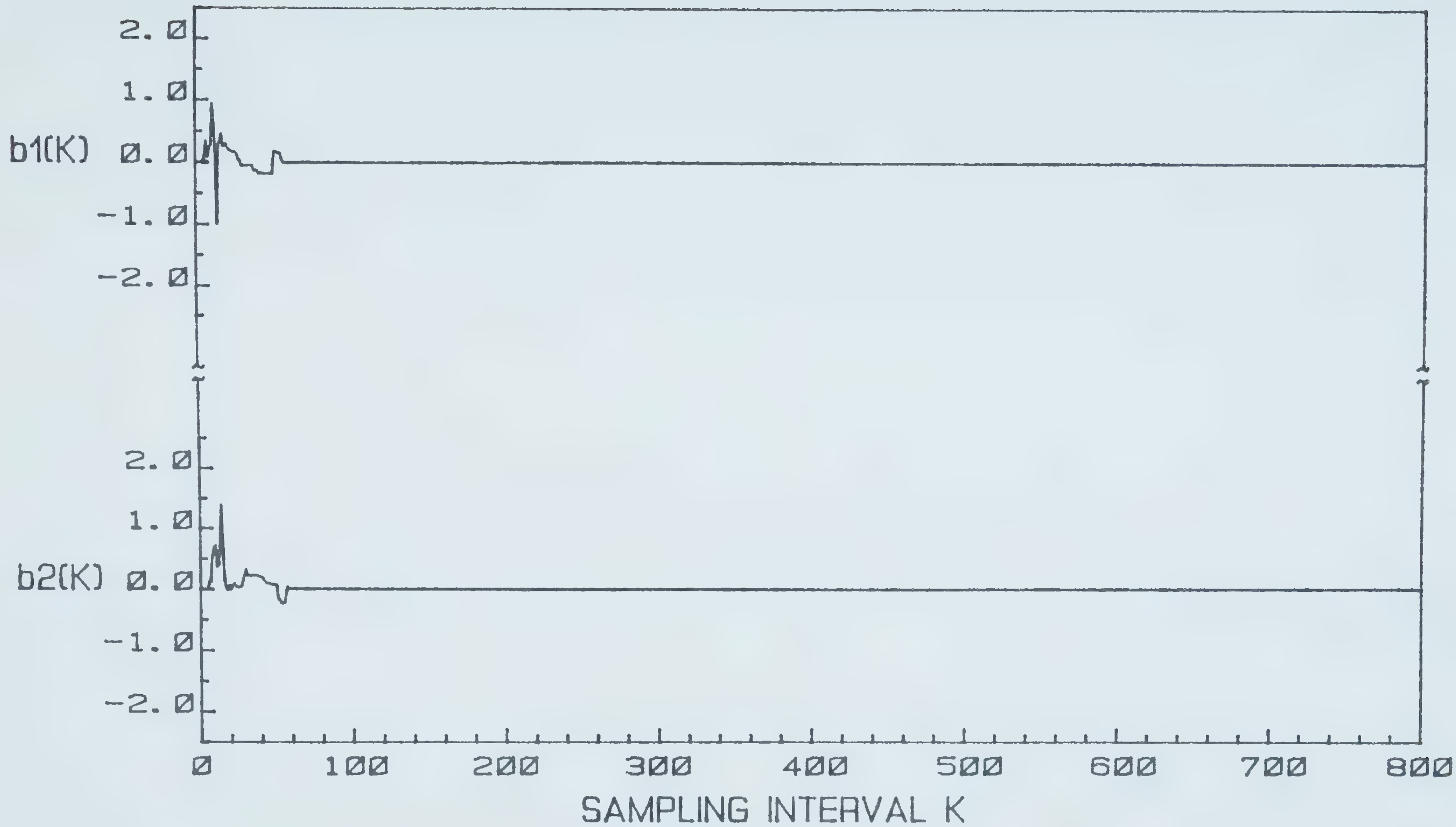

Figure 4.14a. Parameter convergence of adaptive PID controller
(De=2/Da(0<K<500)=2/Da(K>500)=5/M=5/W=0.9/So=1/Po=1.0E+06)

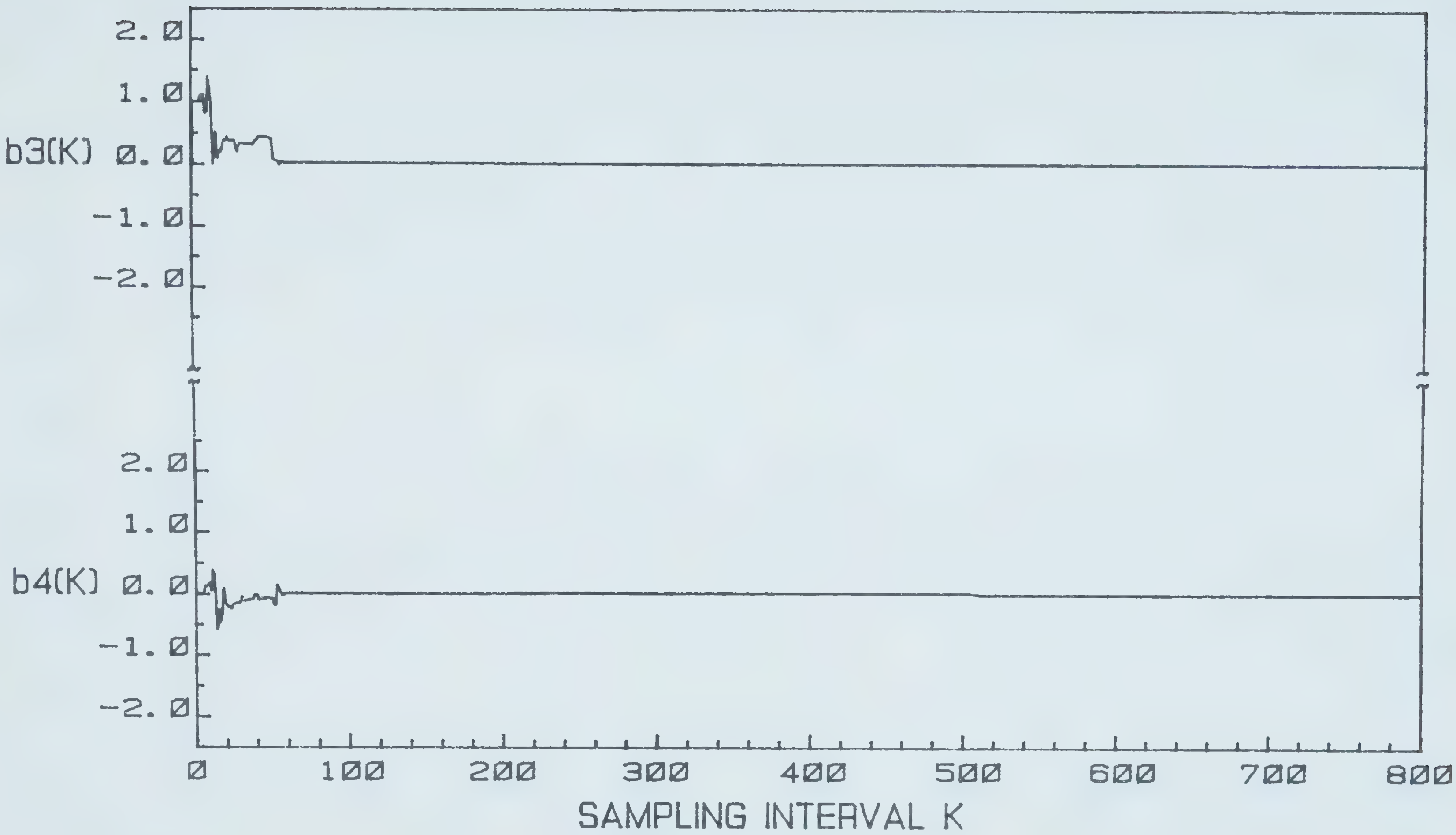

Figure 4.14b. Parameter convergence of adaptive PID controller
(De=2/Da(0<K<500)=2/Da(K>500)=5/M=5/W=0.9/So=1/Po=1.0E+06)

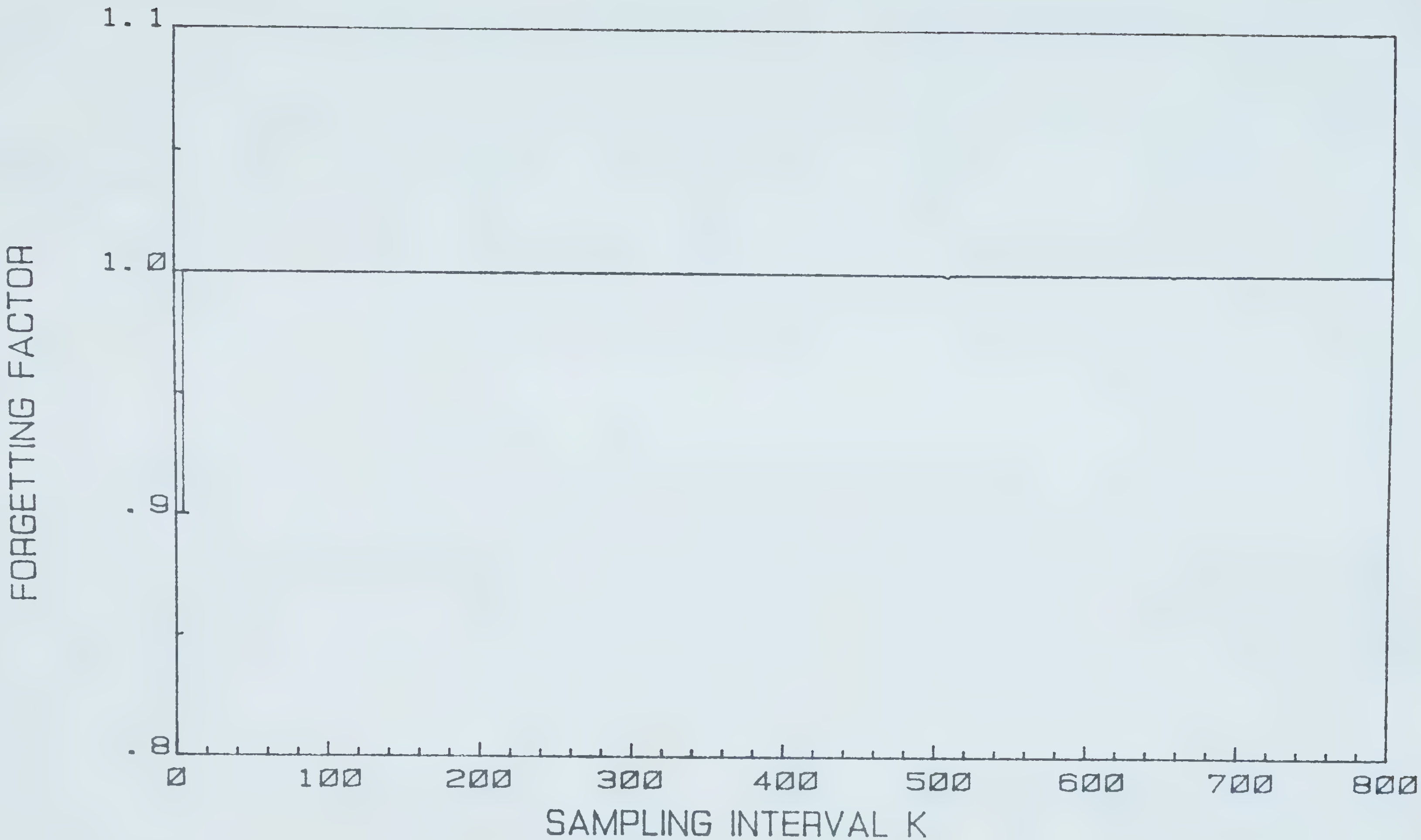

Figure 4.15. Forgetting factor of adaptive PID controller

(De=2/Da(0<K<500)=2/Da(K>500)=5/M=5/W=0.9/So=1/Po=1.0E+06)

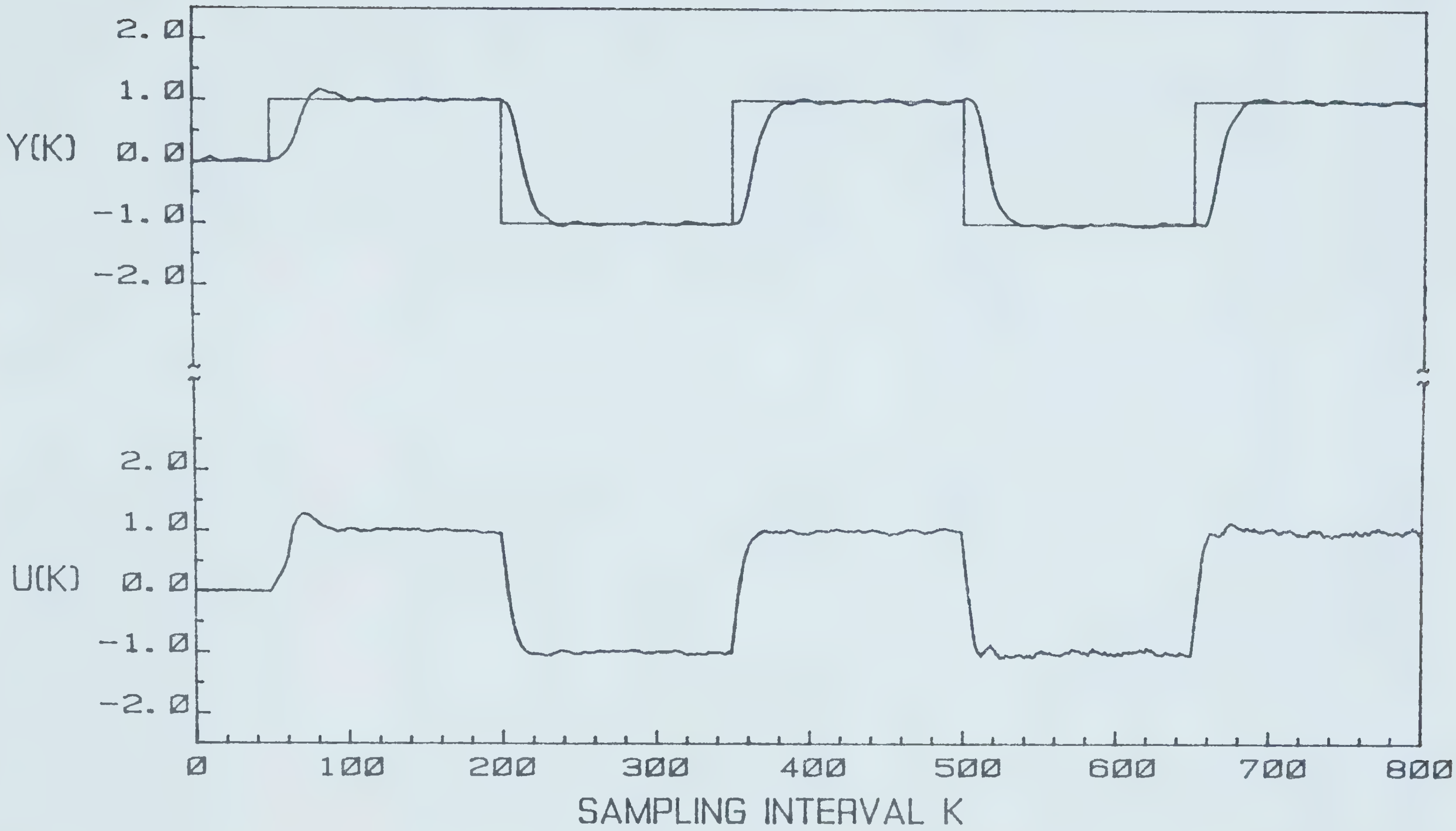

Figure 4.16. Simulated servo response using adaptive PID controller (De=5/Da(0<K<500)=2/Da(K>500)=5/M=8/W=0.9/So=1/Po=1.0E+06)

Table 4.2. List of Parameter Estimates When System Time Delay is Unknown and Varying

Parameter	'True' Parameters	Estimated Parameters	
		k<500	k>500
a_1	-1.5352	-1.5	-1.7
a_2	0.5866	0.56	0.75
b_1	0.0	-0.0022	-0.0053
b_2	0.0	0.0031	0.0057
b_3	0.028	0.024	0.017
b_4	0.0234	0.031	0.011
Time delay for k<500 = 2 k>500 = 5 Time delay was changed at k=500 and T_s=1			

Figure 4.16 gives the system response when the number of extra parameters is made equal to the largest expected time delay. The resulting performance is good but computationally it is less attractive than the case shown in Figure 4.12. Though a maximum expected time delay is required to implement the adaptive PID controller, it is seen in the test runs that the number of extra parameters in polynomial $\hat{B}(z^{-1})$ can be less than the maximum expected time delay.

4.5 Disturbances and Changing Process Gain

The combined closed-loop response for tracking and regulation is shown in Figure 4.17 without feedforward compensation. Unit step changes in setpoint are introduced at k=50, k=200 and k=650. In addition, step load

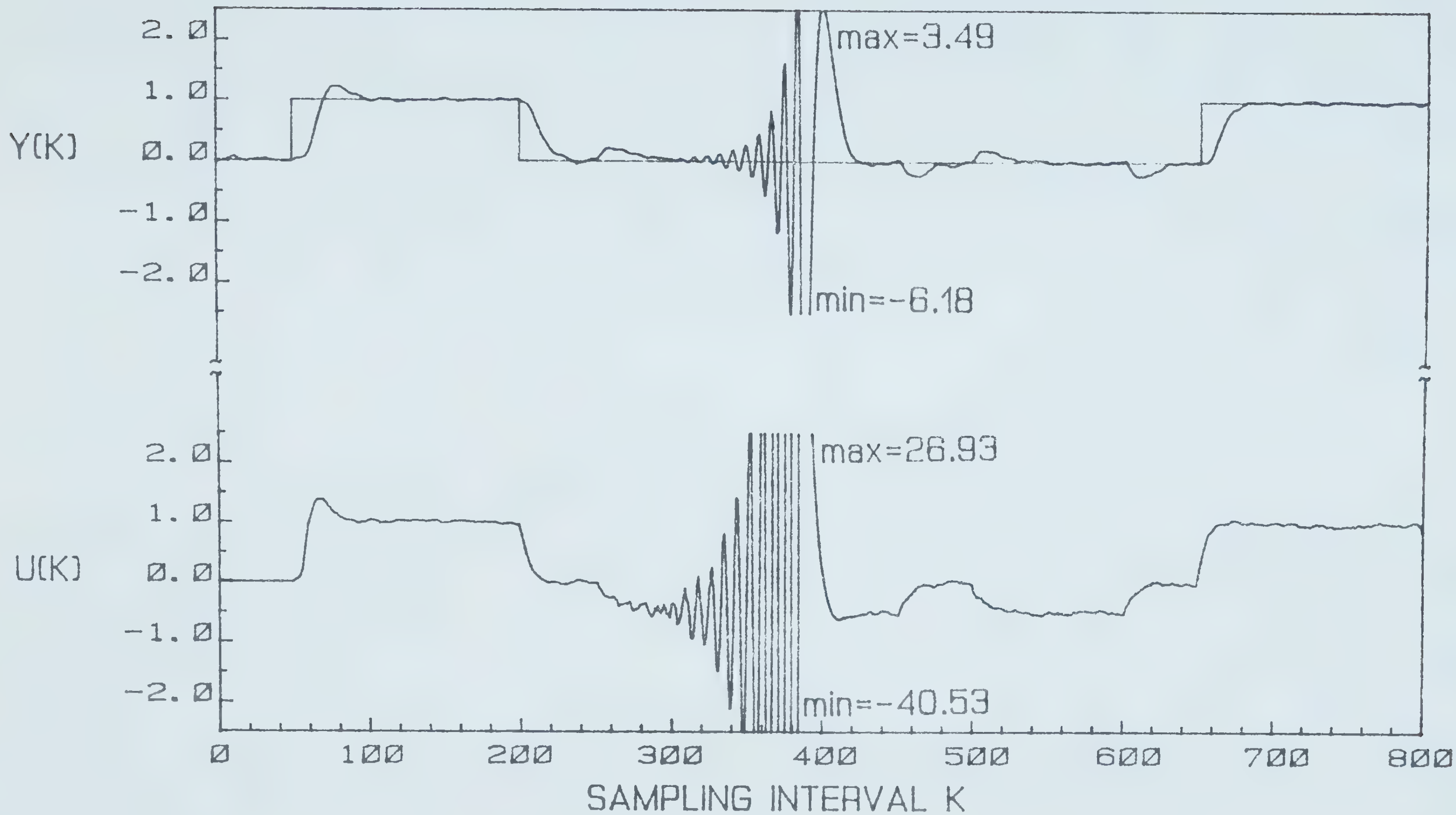

Figure 4.17. Simulated servo & regulatory response using adaptive PID controller w.o.ffd. (Da=De=2/V(250<K<450&500<K<600)=0.5)

disturbances of magnitude equal to 0.5 are introduced at k=250 and k=500 for a period of 200 and 100 sampling intervals respectively.

Figure 4.17 shows oscillatory response from k=320 to k=420. Since the controller is initially tuned to track setpoint alone, the introduction of first load disturbance changes the estimated parameters. The oscillations are therefore the consequence of this initial adaptation period. Moreover, the output response settles down at the desired trajectory before the step load disturbance is taken away. The oscillations are therefore not to be mistaken as the result of the second disturbance. When the step load disturbance is taken away at k=450, it represents a negative step load disturbance to the system. However, it can be seen from Figure 4.17 that the disturbance rejection action is very fast after the controller is tuned in. The second setpoint change is made to test the controller performance at setpoint tracking after it has tuned itself for disturbance rejection.

In Figure 4.18 adaptive feedforward control is added to the adaptive feedback PID controller for the same operating conditions as in Figure 4.17. For feedback plus feedforward control, two additional parameters are estimated to model the measurable disturbance. The addition of feedforward compensation eliminates the oscillations and provides faster response than the one with feedback control alone.

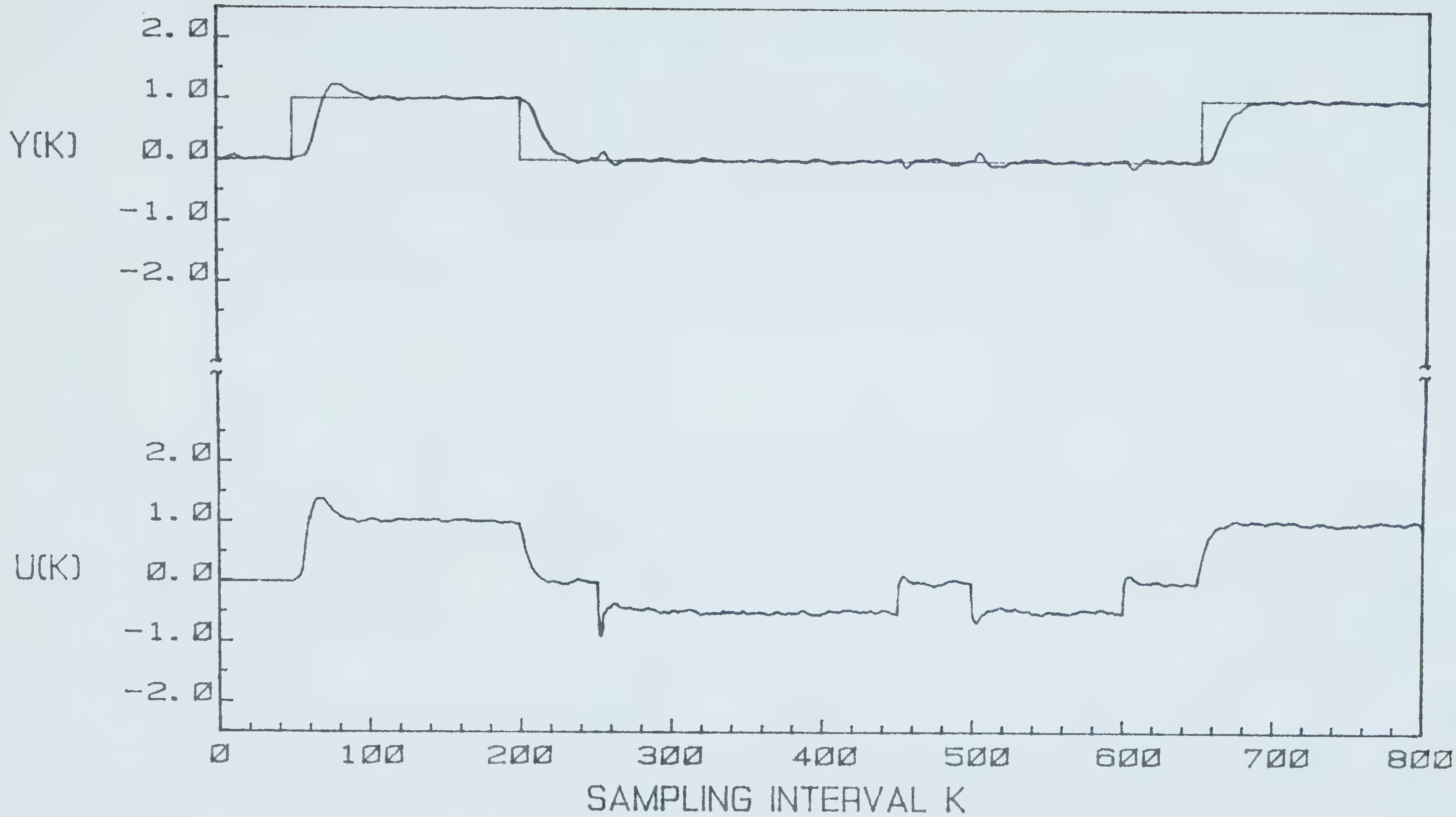

Figure 4.18. Simulated servo & regulatory response using adaptive PID controller w.ffd. (Da=De=2/V(250<K<450&500<K<600)=0.5)

Figure 4.19 shows the case when the process gain is changed from 1 to 2 at k=200. Step load disturbances of magnitude equal to 0.5 are again introduced at k=300 and k=450 for a period of 100 and 350 sampling intervals respectively. Unlike load disturbance, the changing of process gain changes the 'actual' process parameters. Since the estimator has found a set of convergence values for the initial process during the first step change, the introduction of a different process gain introduces a new set of system parameters. Consequently the estimation error increases, and the forgetting factor decreases to a lower value such that the estimator can discount the old data and put heavier weighting on the new information to adapt to the 'new' system. As shown in Figure 4.20, the forgetting factor reaches the lower limit only when the estimation error becomes large, and not at the instant when the sudden change in the process gain occurs. This is expected since the forgetting factor is determined based on the estimation error rather than the process gain. During the adaptation period, the output response becomes very oscillatory. Nevertheless, because of this excitation in the system which gives 'rich' information to the estimator, the controller is well tuned and the system response to the step load disturbances is quickly arrested.

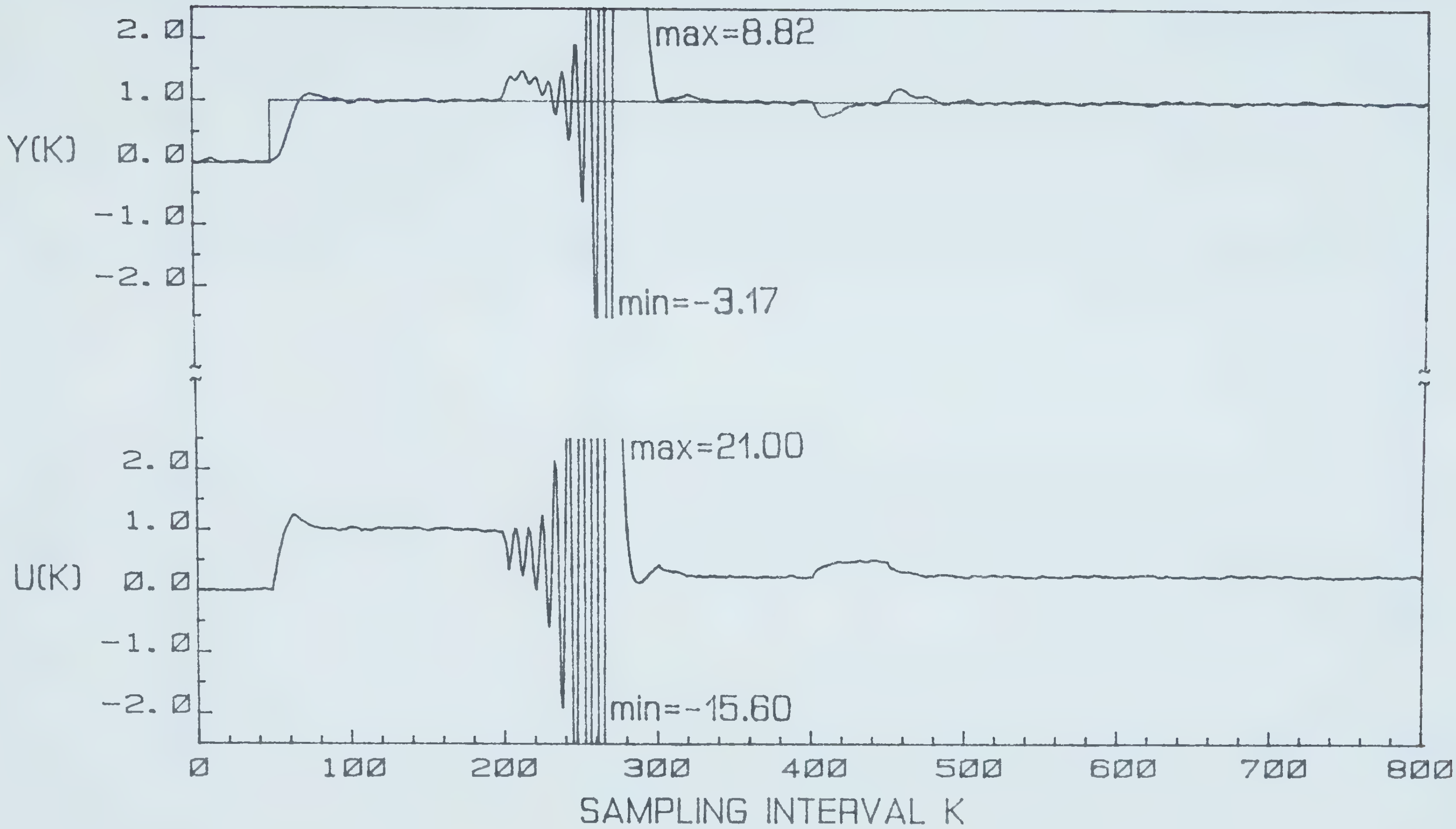

Figure 4.19. Simulated servo & regulatory response using adaptive PID controller w.o.ffd. (Da=De=2/G(K>200)=2/V(300<K<400&450<K<800)=0.5)

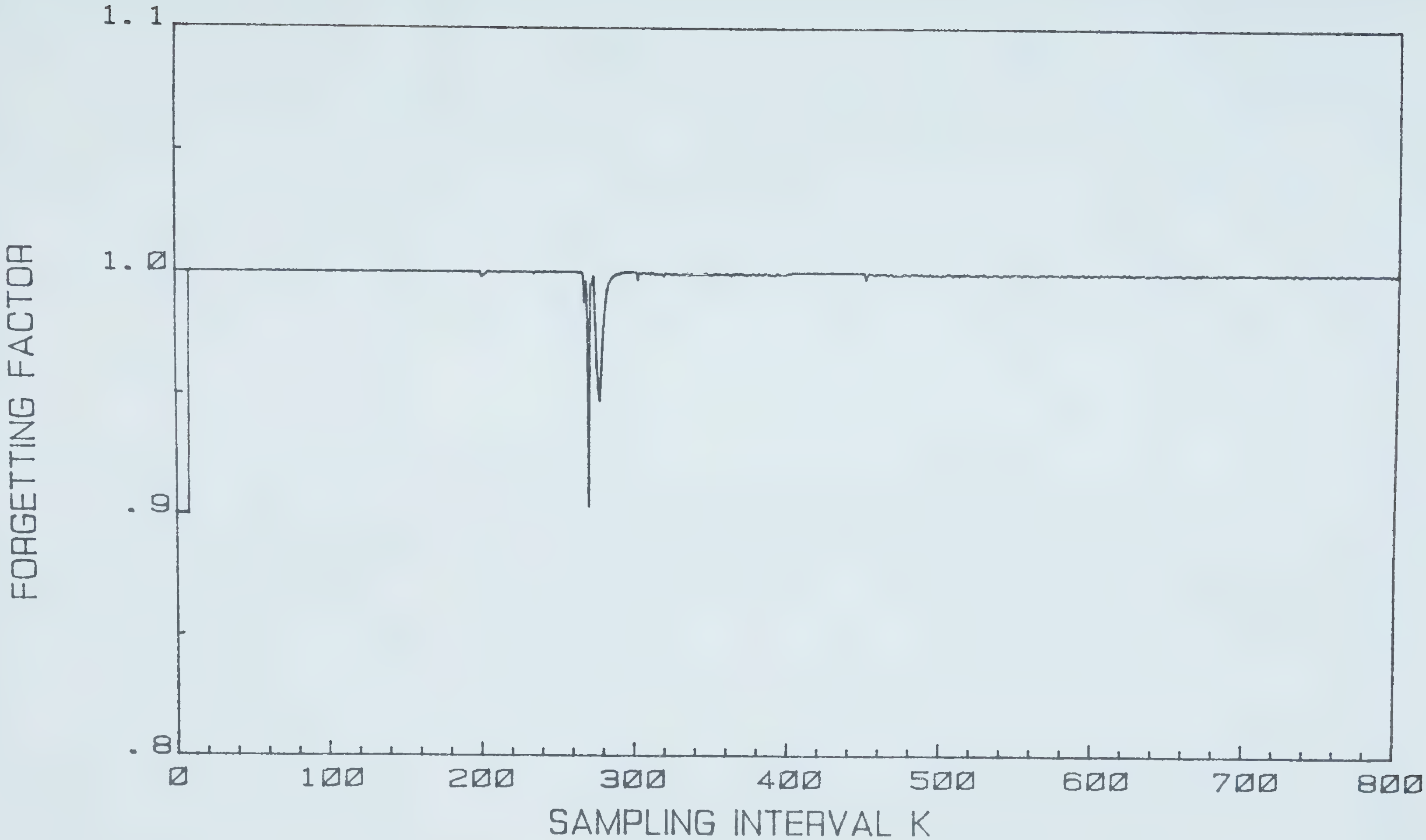

Figure 4.20. Forgetting factor of adaptive PID controller
(Da=De=2/G(K>200)=2/V(300<K<400&450<K<800)=0.5)

4.6 Delay Dominated Systems

To evaluate the performance of the adaptive PID controller on a delay dominated system, i.e. when $d/\tau > 1$, the time delay in the simulation model of equation (4.2) is increased from the previous maximum of $5T_s$ to $8T_s$. The adaptive PID controller is then applied to this system with 4 extra parameters in polynomial $\hat{B}(z^{-1})$. The polynomial $\hat{B}(z^{-1})$ is therefore underparameterized by 4 parameters. Figure 4.21 shows the result obtained for this application. Due to the large time delay in the system, the initial adaptation period is longer and the initial output response is oscillatory with high overshoot. However, it can be seen from Figure 4.21 that the output response tracks the setpoint very well after this initial tuning period. The initial oscillatory response can be avoided if the initial system identification is done in the background while the process is operating under a fixed gain controller.

4.7 Summary

This chapter discusses the results obtained from the simulation studies on a 'bench-mark' example, and serves as a stepping stone to the experimental studies presented in the next chapter. The evaluation procedures are categorized into: i) systems with known and constant time delays; ii) systems with unknown but constant time delays; iii) systems with unknown and varying time delays; iv) load disturbances and systems with changing process gains; v) delay dominated

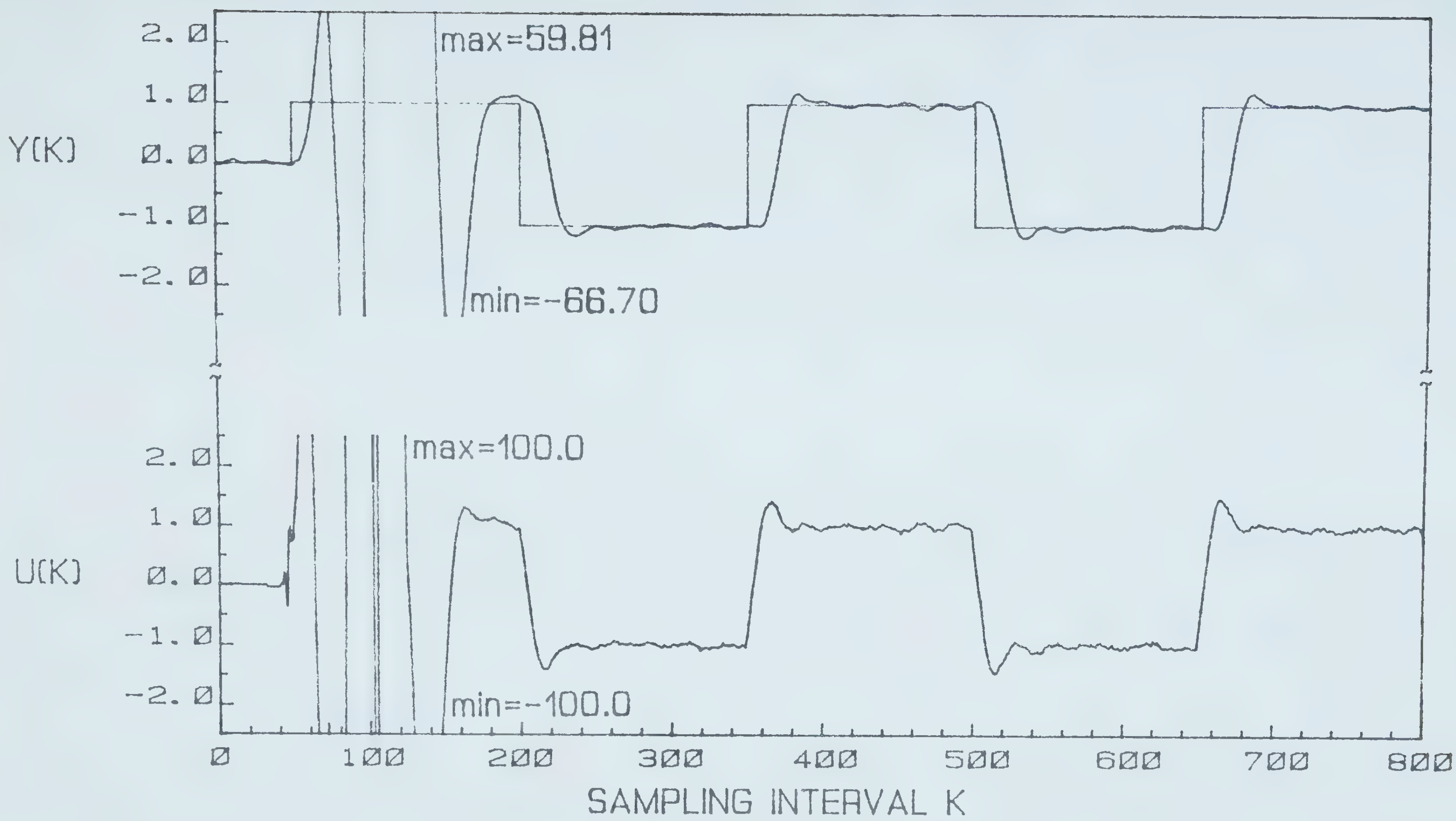

Figure 4.21. Simulated servo response using adaptive PID controller
(De=4/Da=8/M=7/W=0.9/So=1/Po=1.0E+06)

systems. The discussions focus mainly on the choices of several initial parameters in order to start the algorithm, and the effects of these choices on the closed-loop asymptotic tracking and regulatory properties of the adaptive PID controller. The initial choices of the covariance matrix and the non-zero b parameter are found to be non-critical, i.e. P(0) can be set to a relatively large identity matrix and the non-zero b parameter can be any one of the b parameters. Though not sensitive, the tuning parameter w_1 is found to give best performance when a higher value is chosen, and a relatively large value of Σ_0 should be chosen to prevent the forgetting factor from remaining at the lower limit. Also, the total number of extra parameters in polynomial $\hat{B}(z^{-1})$ to handle time delay systems needs not be the same as the maximum expected time delay in the system. This property is particularly useful when dealing with systems with unknown and varying time delays. It reduces a lot of computational time when time delay is large. The adaptive feedforward compensator is also studied and found to improve the output performance considerably. Finally, the adaptive PID controller is found to work well even when it is applied to a delay dominated system.

5. Experimental Evaluation

5.1 Introduction

To evaluate the adaptive PID controller performance on a real process, the algorithm was implemented on a HP-1000 digital computer to control the temperature of a continuous stirred-tank heater. This chapter outlines the process equipment and control hardware that were used to experimentally evaluate the adaptive PID control algorithm and discusses the results obtained.

5.2 Description of Equipment

The continuous stirred-tank heater used in this study is located in Room 274B of the Chemical-Mineral Engineering Building at the University of Alberta. A detailed description of the equipment is also available in Lieuson, Morris, Nazer and Wood [1980] and Thesen [1981]. A schematic diagram of the equipment is shown in Figure 5.1. The tank is 15 cm in diameter and 55 cm in height. The temperature of city water used as the inlet cold water ranges from 11°C to 18°C depending upon the outdoor temperature and is measured by thermocouple A. Since city water is used in the entire building for other process equipment and the inlet flowrate is merely monitored by a hand valve, fluctuations on the inlet flowrate are inevitable. Inlet flowrate, measured by an orifice/mercury manometer, is maintained at 8.2 kg/min during steady-state operations. Parallel to the manometer is

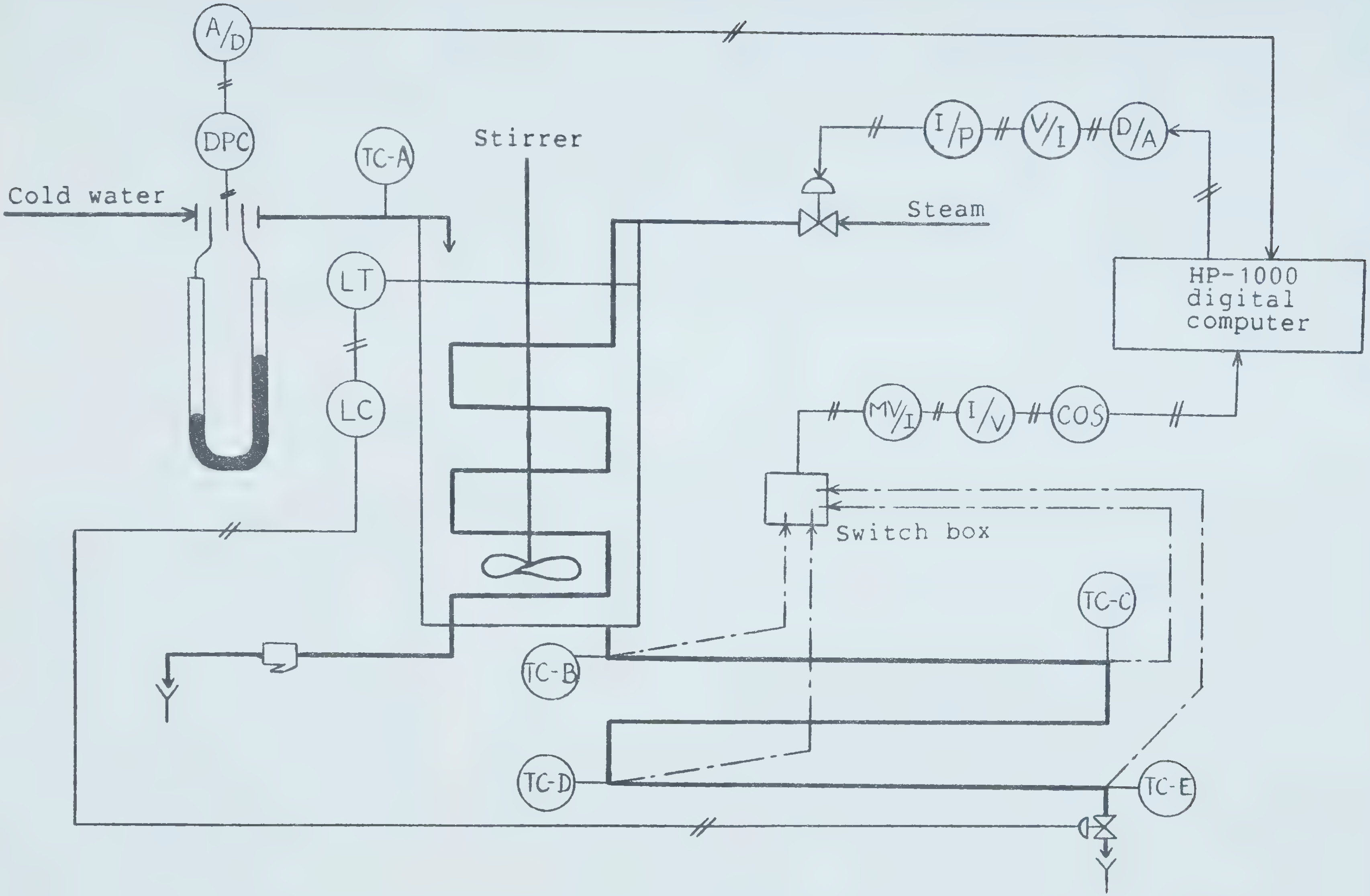

Figure 5.1 Schematic diagram of the stirred-tank heater.

a differential pressure cell which provides the flowrate measurement signal to the computer.

A constant holdup of approximately 8.2 kg is maintained in the tank by a level controller. The inlet cold water is heated by saturated steam at a pressure of 50 psig. The pneumatic control valve on the steam line is an equal percentage type and thus gives rise to a non-linear relationship between the input air pressure and the steam flowrate. The outlet water temperature can be measured by either thermocouple B, C, D or E. The thermocouples, as seen in Figure 5.1, are located at different points on the outlet pipe to deliberately introduce different time delays in the system. Only one thermocouple can be used at a time and is selected through a switch box as shown. A load disturbance is introduced into the system by changing the inlet flowrate. By so doing, it also changes the outlet flowrate to maintain a constant holdup. Changes in outlet flowrate cause the time delay to vary since the major system time delay is the transportation delay.

Except for cases where changing time delays are involved, thermocouple E is used for the experimental study described here. The choice of this sensor location in preference to the others is because it gives the longest time delay. Relationship between the inlet flowrate and the time delay is calibrated for thermocouple E and is given by:

$$d = -2.16\ R + 33.44 \tag{5.1}$$

Where d is the system time delay in seconds and R is the inlet flowrate in kg/min.

The process is interfaced to a HP-1000 digital computer in the Data Acquisition, Control and Simulation (DACS) Centre in the Department of Chemical Engineering at the University of Alberta. The thermocouple signal in millivolts is converted to a current signal of 4 to 20 milliamperes (mA). The corresponding 1 to 5 volt output developed across a resistor is then 'read' by the analog-to-digital (A/D) converter. The control signal from the digital computer, on the other hand, is sent to a current-output-station (COS) which also serves as a zero-order hold device. The output from the COS is a 4 to 20 mA signal. This current signal is then converted to a 3 to 15 psig air pressure signal to position the control valve through a current-to-pressure (I/P) converter.

5.3 Experimental Results

The experimental results obtained by using the adaptive PID controller are compared to those obtained by using conventional discrete, fixed gain, PID controller. No attempts are made to compare the adaptive PID controller performance with the self-tuning PID controller performance proposed earlier in the literature [Wittenmark and Åström,

1980; Isermann, 1981; Corripio and Tompkins, 1981; Gawthrop, 1982; Bànyàsz and Keviczky, 1982; Hetthéssy, Keviczky and Bànyàsz, 1983; Åström and Hägglund, 1983; Cameron and Seborg, 1983; Song, 1983], since the earlier works are not designed to handle unknown and/or varying time delay systems. The major emphases on the evaluation are: to study the controller performance in handling systems with unknown but constant or varying time delay; and to study the disturbance rejection properties of the controller. The results are presented in the following order: setpoint tracking for a) known and constant time delay system b) unknown and constant time delay system c) unknown and changing time delay system; and disturbance rejection with and without feedforward compensation.

As mentioned in the previous chapters, when the process is represented by a first order plus time delay model, the resulting controller structure is equivalent to that of the conventional discrete PI controller. Experimental runs with the adaptive PI controller were also conducted to evaluate its performance.

Initially the process parameter estimation was done in the background while the process was operating under a fixed gain PI controller. This was done to avoid excessive control action during the initial period. When a process is unknown, the initialized parameters generally contain large errors. Since the controller design uses the certainty-equivalence principle, the control algorithm accepts the current

estimates without considering their uncertainty. Consequently, the initial control action might be meaningless. Hard limits on the control valve are placed at 20 and 100% valve opening. The thermocouples are calibrated for 25°C to 55°C. Below 20% valve opening, the temperature in the tank is below the calibrated range. Due to the background estimation, the parameter estimates can be initialized to zero. However, in order to be consistent with the simulation runs, the last two $\hat{b}$ parameters for a second order delay model and the last $\hat{b}$ parameter for a first order delay model are initialized to one. This choice also avoids large initial transients in the control valve. The fixed gain PI controller settings are Kc=3.5%valve/°C and K_i=0.007sec^{-1}.

5.3.1 Constant and Known Time Delay

Figure 5.2 shows the stirred-tank heater response just after being switched from fixed gain PI controller to the adaptive PID controller. The outlet temperature is measured with thermocouple E which also introduces the longest transport delay. With a sampling interval of 4 seconds as used in Thesen [1981], the total delay is estimated to be 4 sampling periods. In the case of known time delay, four additional $\hat{b}$ parameters are estimated. The desired closed-loop system pole is located at z=0.9, the value of Σ_0 is chosen to be 5, and the initial covariance matrix is 10^3I. Convergence of the estimated parameters are shown in

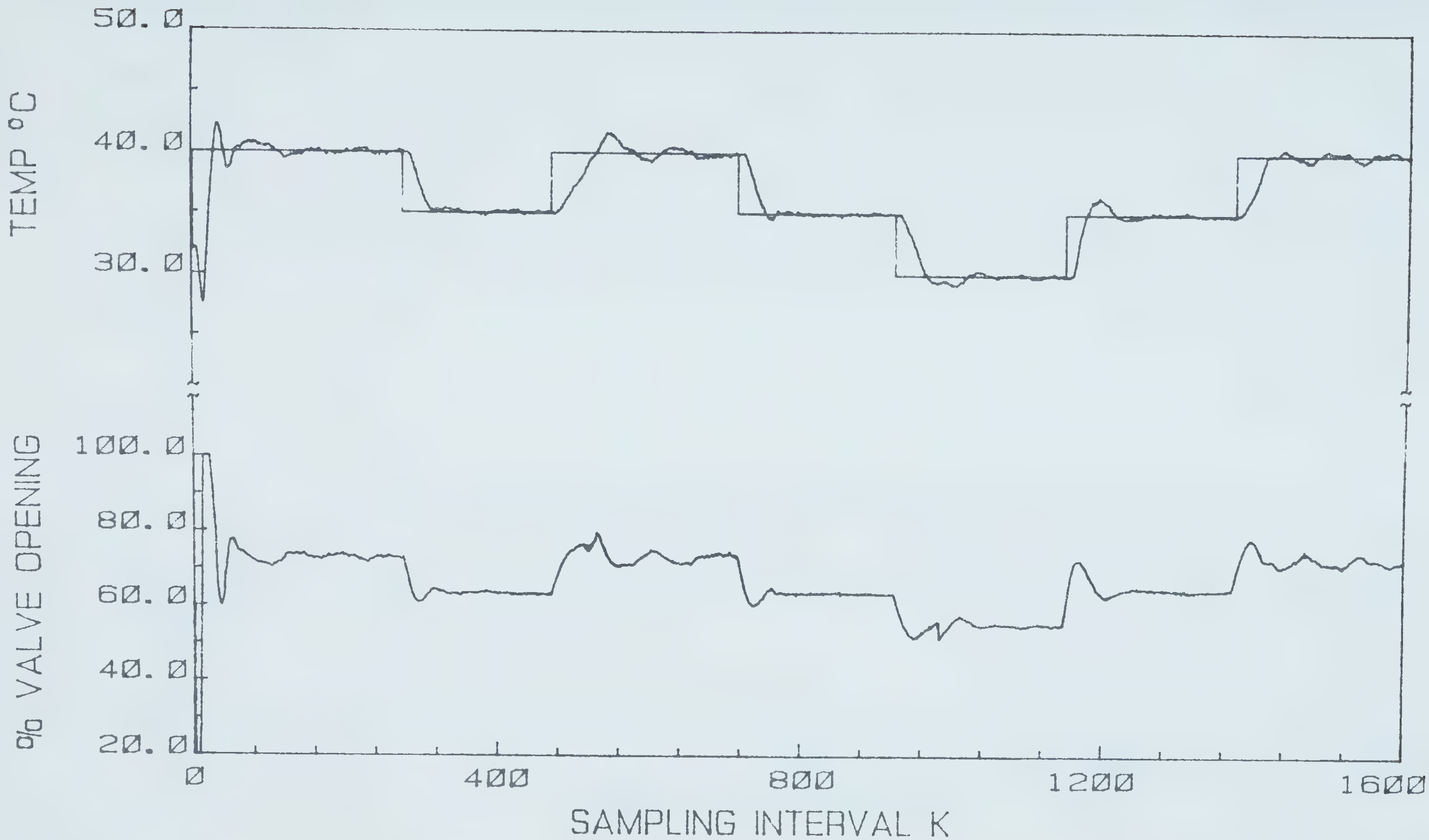

Figure 5.2 Stirred-tank heater response using adaptive PID controller
(Da=De=4/W=0.9/So=5)

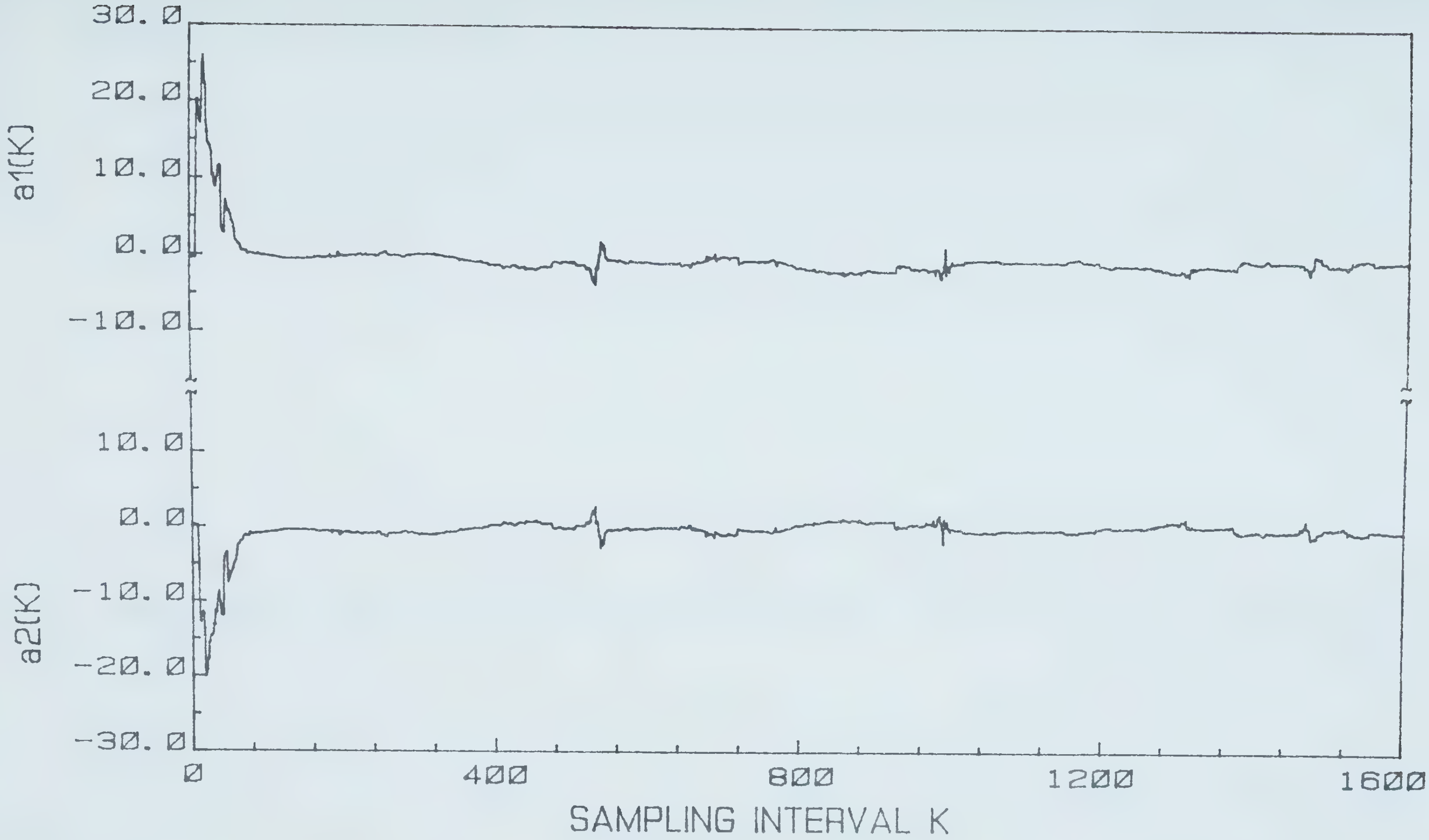

Figure 5.3a. Parameter convergence of adaptive PID controller (Da=De=4/W=0.9/So=5)

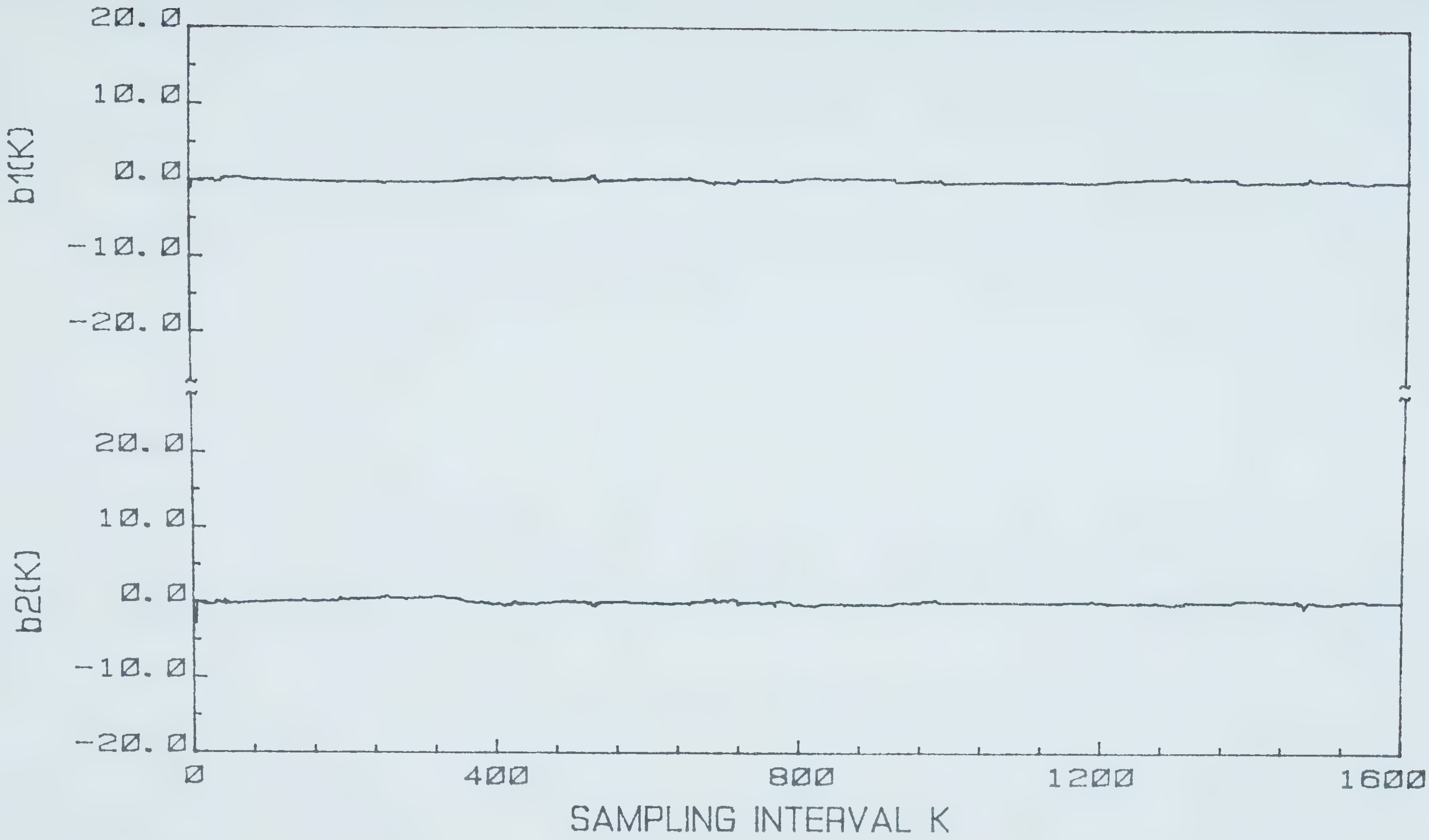

Figure 5.3b. Parameter convergence of adaptive PID controller

(Da=De=4/W=0.9/So=5)

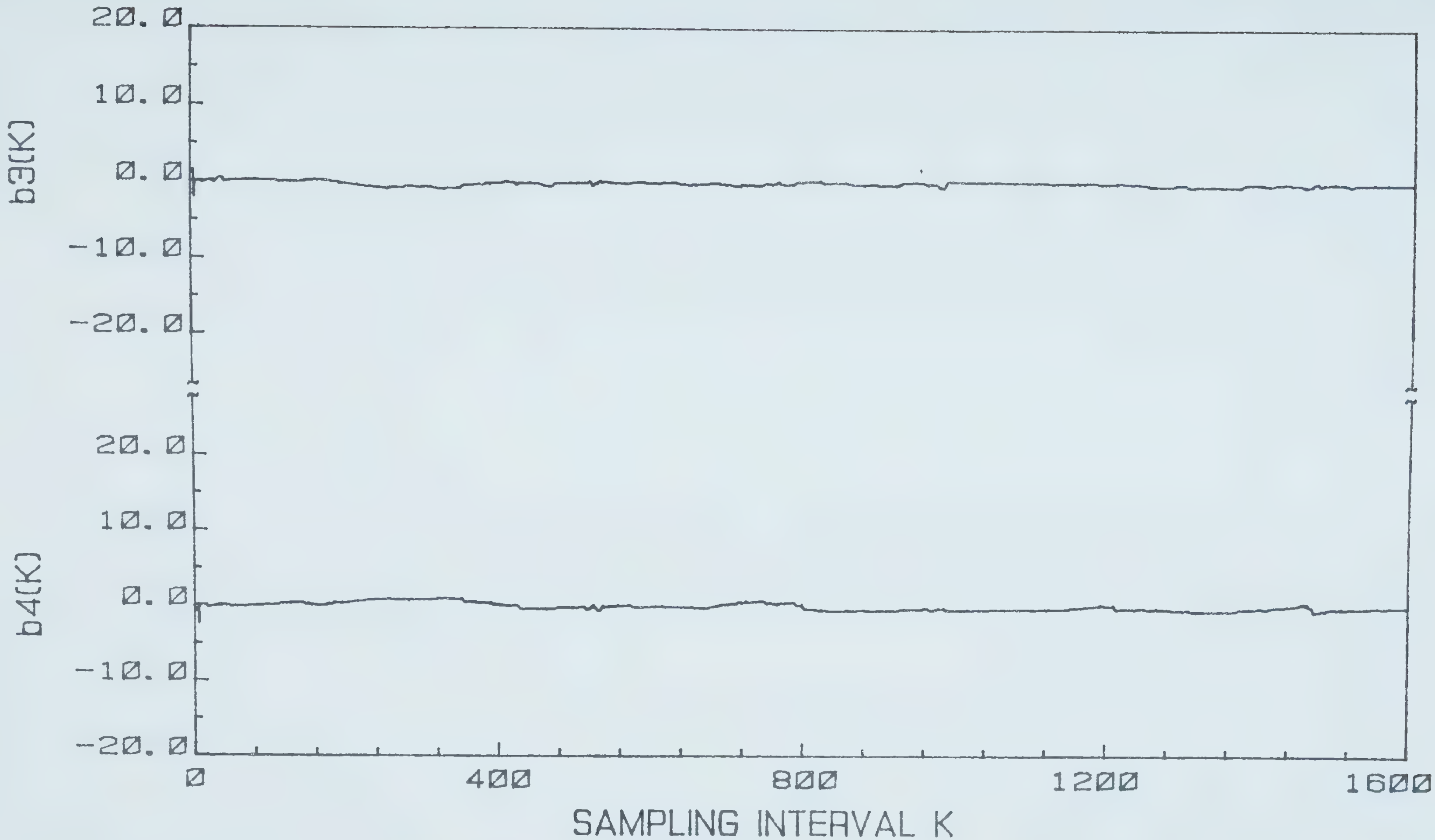

Figure 5.3c. Parameter convergence of adaptive PID controller (Da=De=4/W=0.9/So=5)

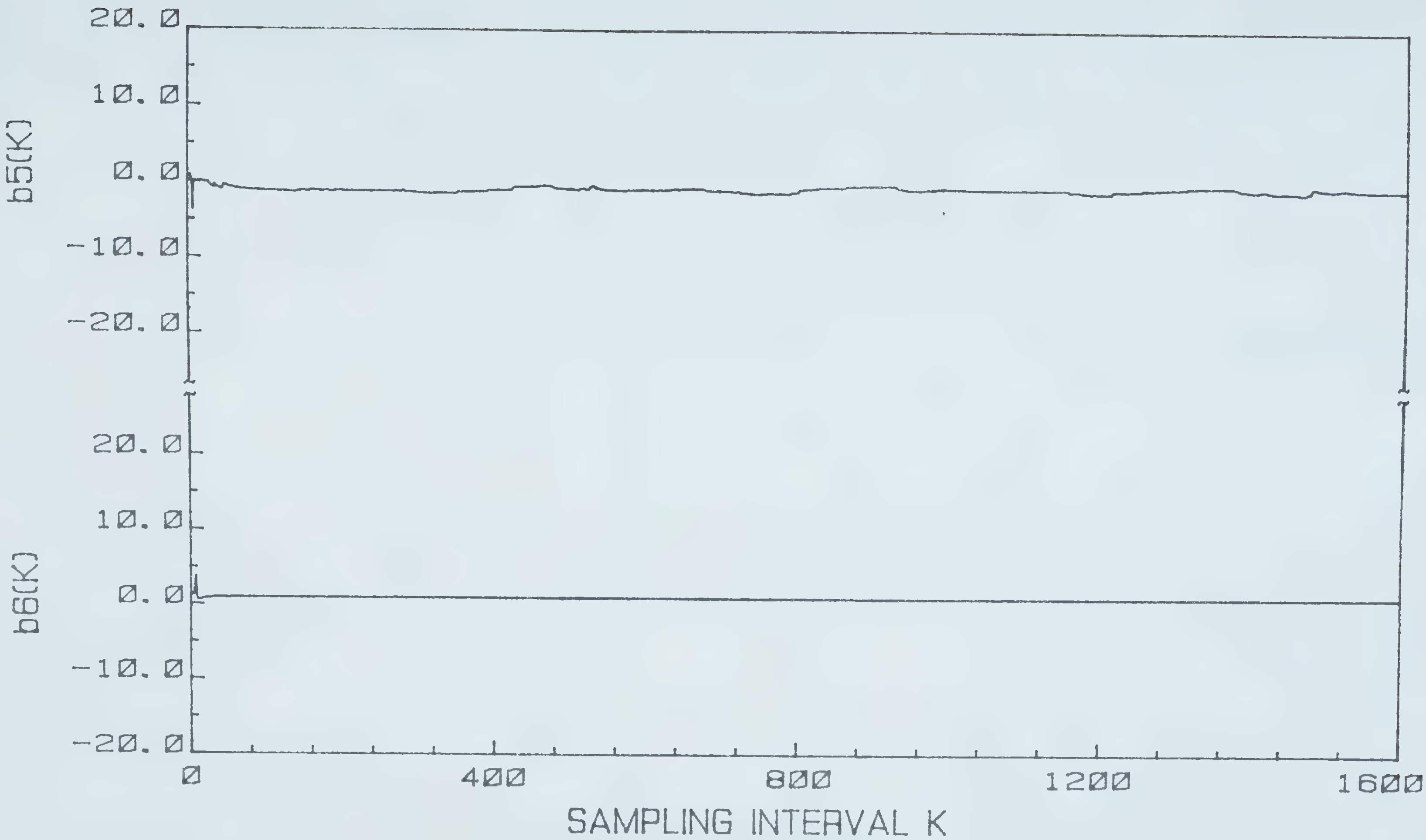

Figure 5.3d. Parameter convergence of adaptive PID controller
(Da=De=4/W=0.9/So=5)

Figure 5.3a, 5.3b, 5.3c and 5.3d. Though the parameters converged to constant values during the fixed gain PI controller period of operation, they fluctuate again at the time intervals related to the setpoint changes. This is expected since the adaptive controller settings are determined from the parameter estimates and the setpoint change has served to reactivate the estimator. The fluctuation during the first setpoint change is the largest and then the changes become smaller at each subsequent setpoint changes. This implies that parameter convergence has eventually occurred.

The adaptive PID controller settings are plotted in Figure 5.4a, 5.4b and 5.4c. Small changes in the parameter estimates lead to large jumps in the controller settings as shown around k=400, k=800 and k=1280. These changes in the estimates can be explained as a result of the choice of Σ_0 and consequently the value of forgetting factor. As seen from Figure 5.5, the large spikes in the controller settings occur when the forgetting factor stays near the lower limit for some time. When the forgetting factor is less than 1 for a period of time, it increases not only the covariance matrix but also makes the estimator adapt faster to the changes in the system. When this happens while there are no changes in the system, it can lead to oscillatory response and, in the worst case, an unstable system. On the other hand, this also indicates that Σ_0 should be increased to a higher value to prevent the forgetting factor from remaining

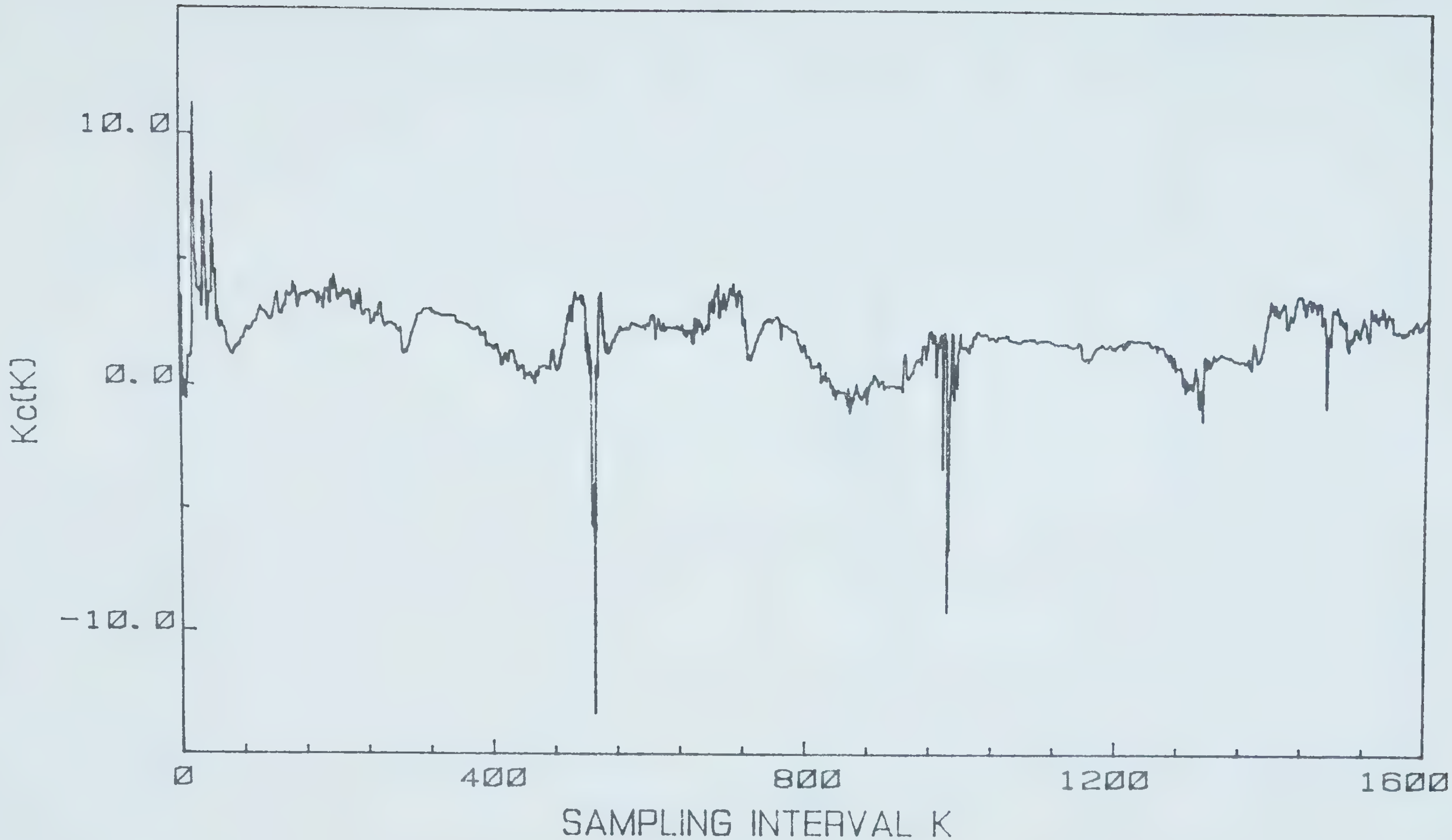

Figure 5.4a. Controller settings of adaptive PID controller

(Da=De=4/W=0.9/So=5)

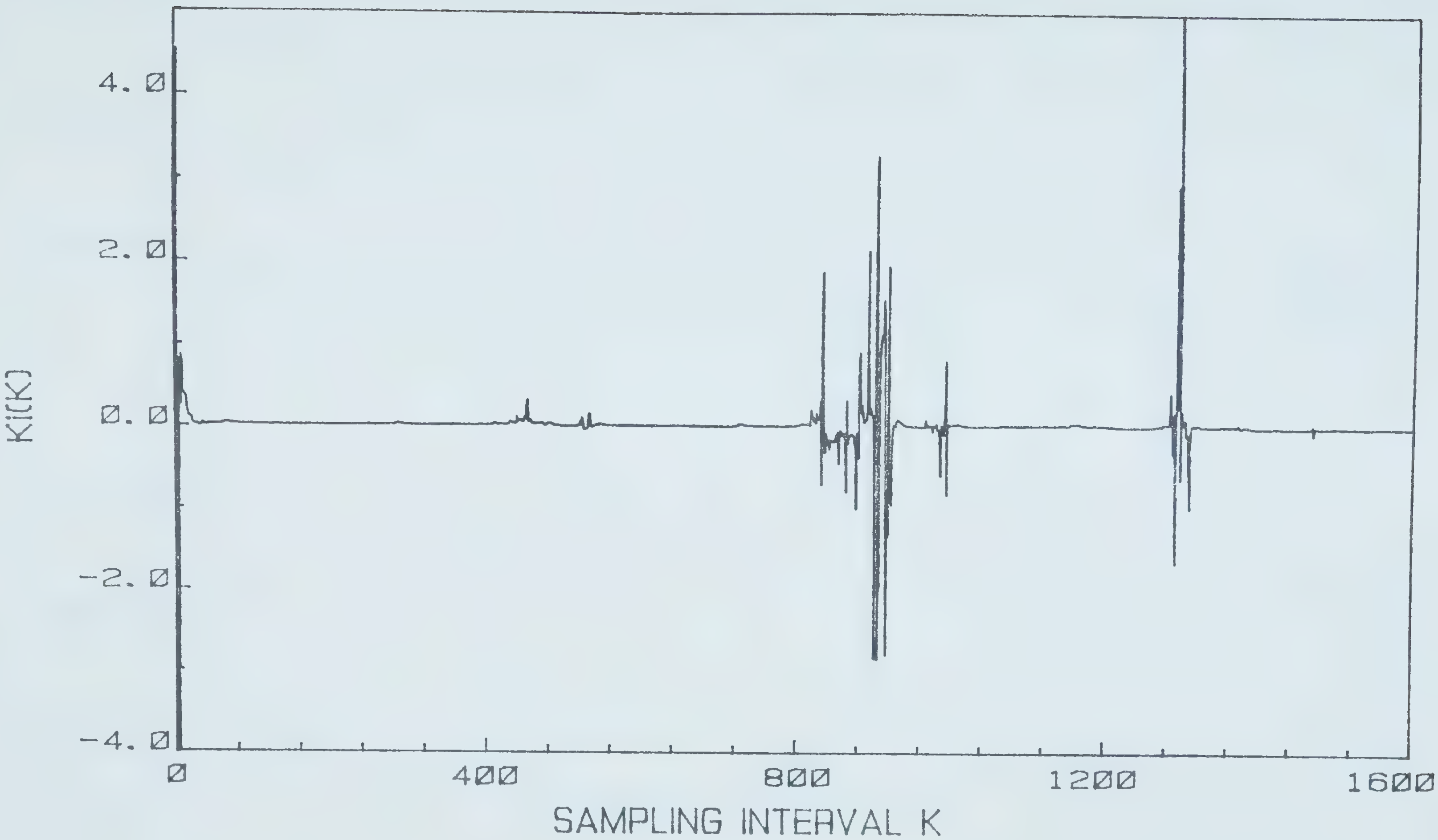

Figure 5.4b. Controller settings of adaptive PID controller
(Da=De=4/W=0.9/So=5)

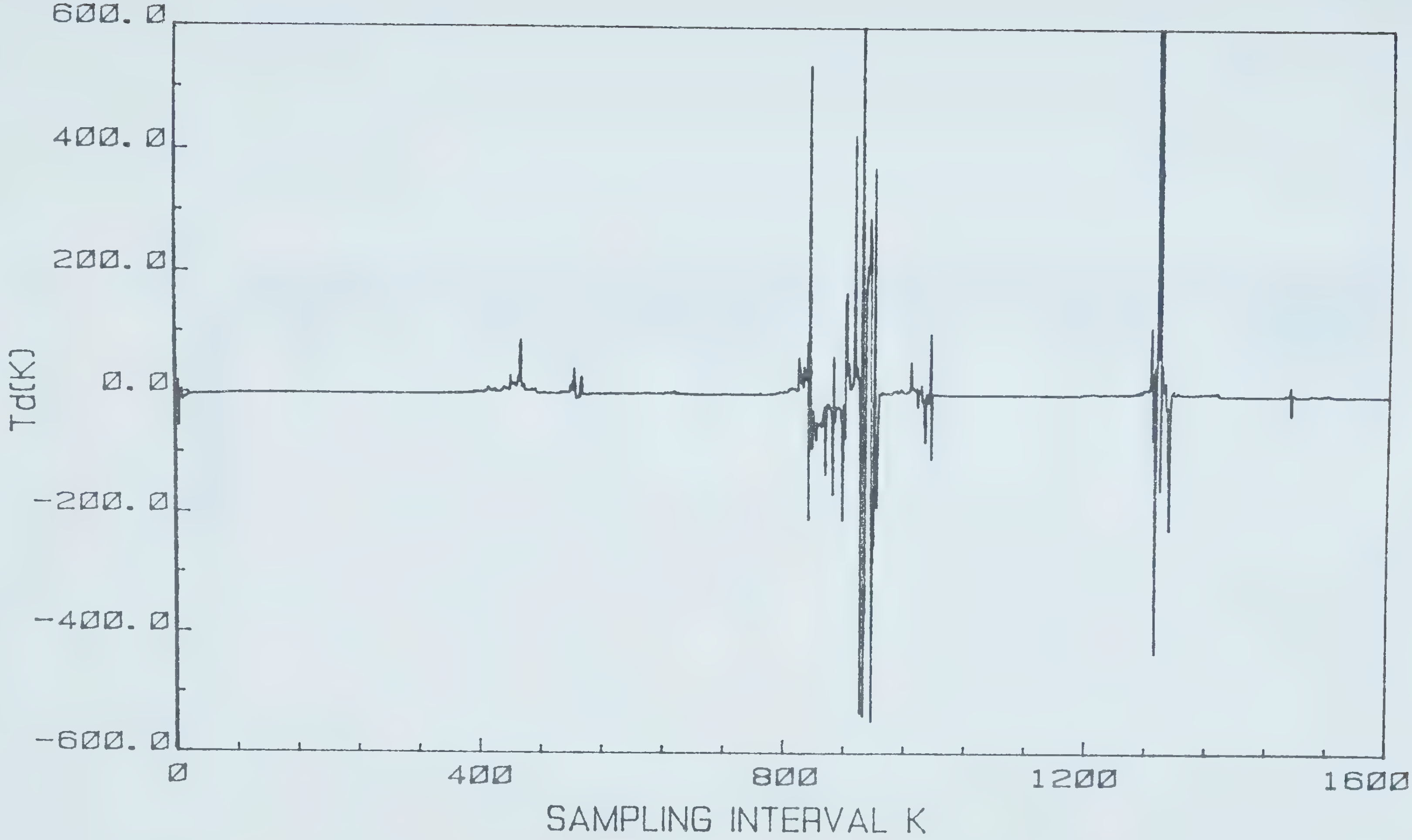

Figure 5.4c. Controller settings of adaptive PID controller
(Da=De=4/W=0.9/So=5)

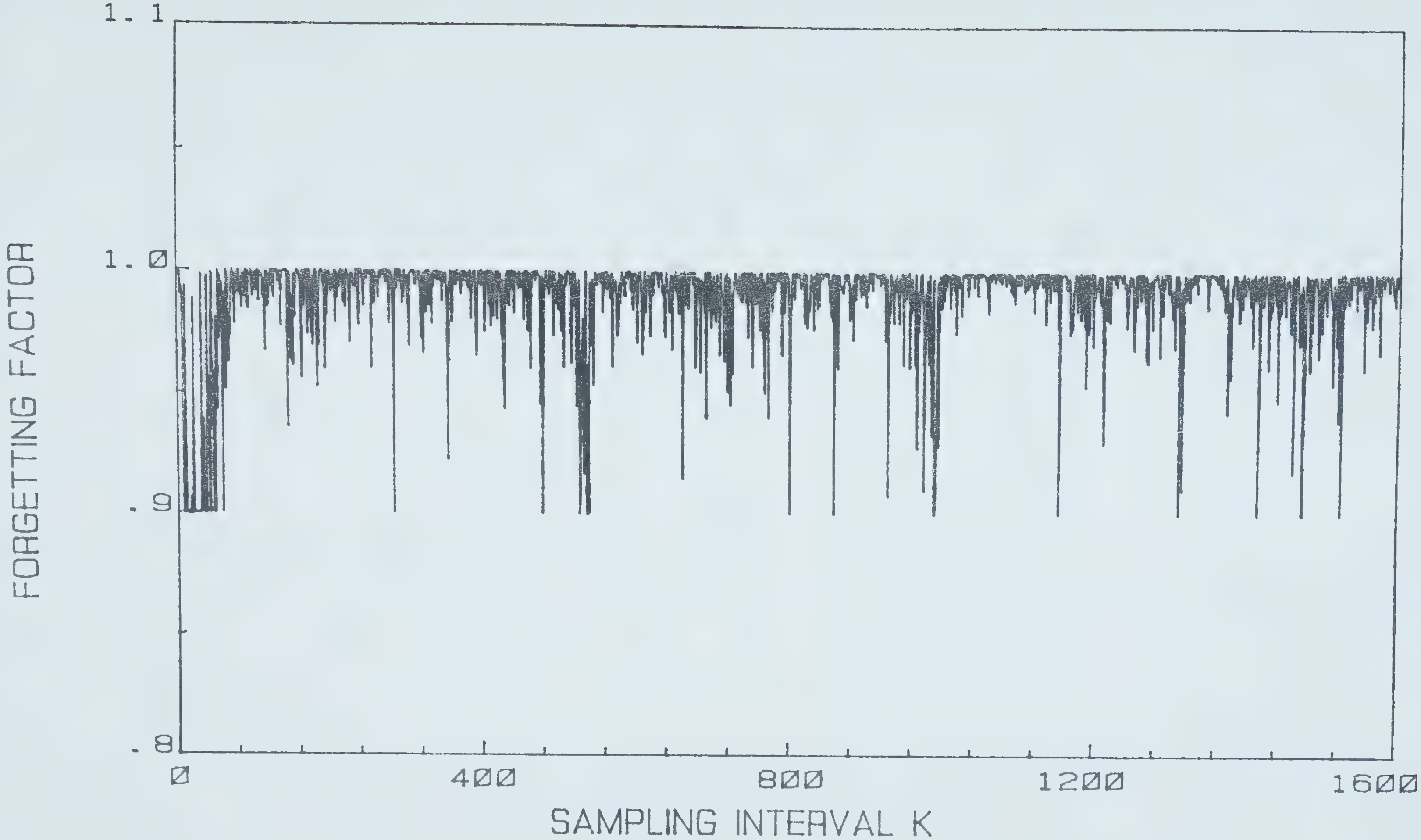

Figure 5.5. Forgetting factor of adaptive PID controller
(Da=De=4/W=0.9/So=5)

at the lower limit when there are no changes in the system (cf. Figure 5.24). It should be emphasized that the choice of Σ_0 also depends on the initial parameter estimates obtained under the control of fixed gain controller. This run is chosen to illustrate the tuning effect of Σ_0. Above all, it is chosen to show the 'robustness' of the adaptive PID controller.

Under similar process operating conditions as in Figure 5.2, adaptive PI controller is used to control the outlet temperature of the stirred-tank heater. Comparable results to Figure 5.2 are obtained and are given in Figure 5.6. The resulting controller settings are shown in Figure 5.7a and Figure 5.7b.

Fixed gain discrete PID controller is used as a basis to compare the performance of the adaptive PID controller. The stirred-tank heater response using fixed gain PID controller is shown in Figure 5.8. The controller settings are estimated by IAE technique [Miller, Lopez, Smith and Murrill, 1967] and after fine tuning, the values are found to be $Kc=5\%valve/°C$, $K_i=0.005sec^{-1}$ and $T_d=10sec$. Better performance by both adaptive PI and PID controllers is evident. Fixed gain PID takes a longer time to reach steady-state and gives higher overshoots with longer rise time. Theoretically, the faster response given by both adaptive PID and PI controllers is due to the summing effect of the $\hat{b}$ parameters. By replacing the polynomial $B(z^{-1})$ of equation (2.24) with $\Sigma\hat{b}_i$, the response is different in the

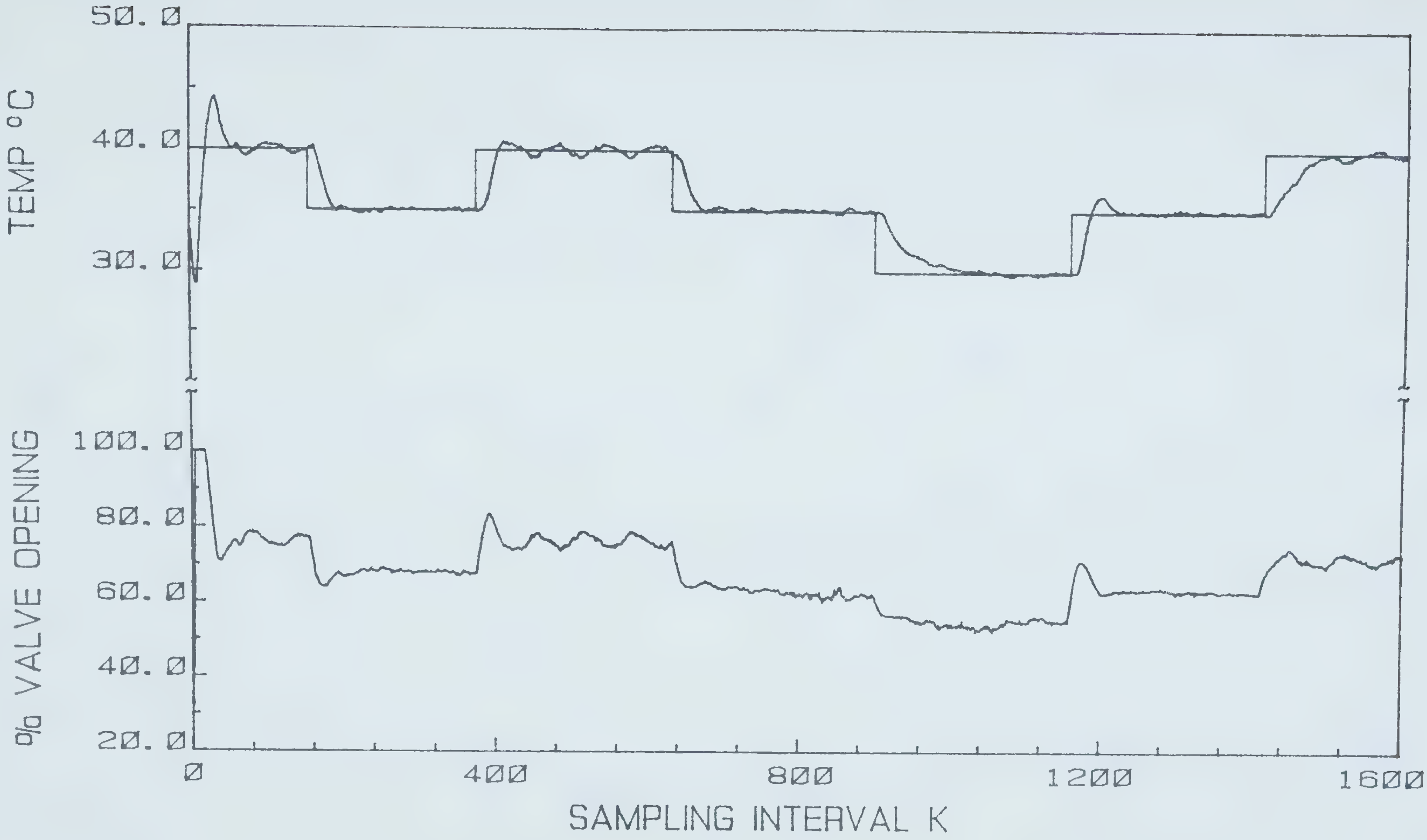

Figure 5.6. Stirred-tank heater response using adaptive PI controller (Da=De=4/W=0.9/So=5)

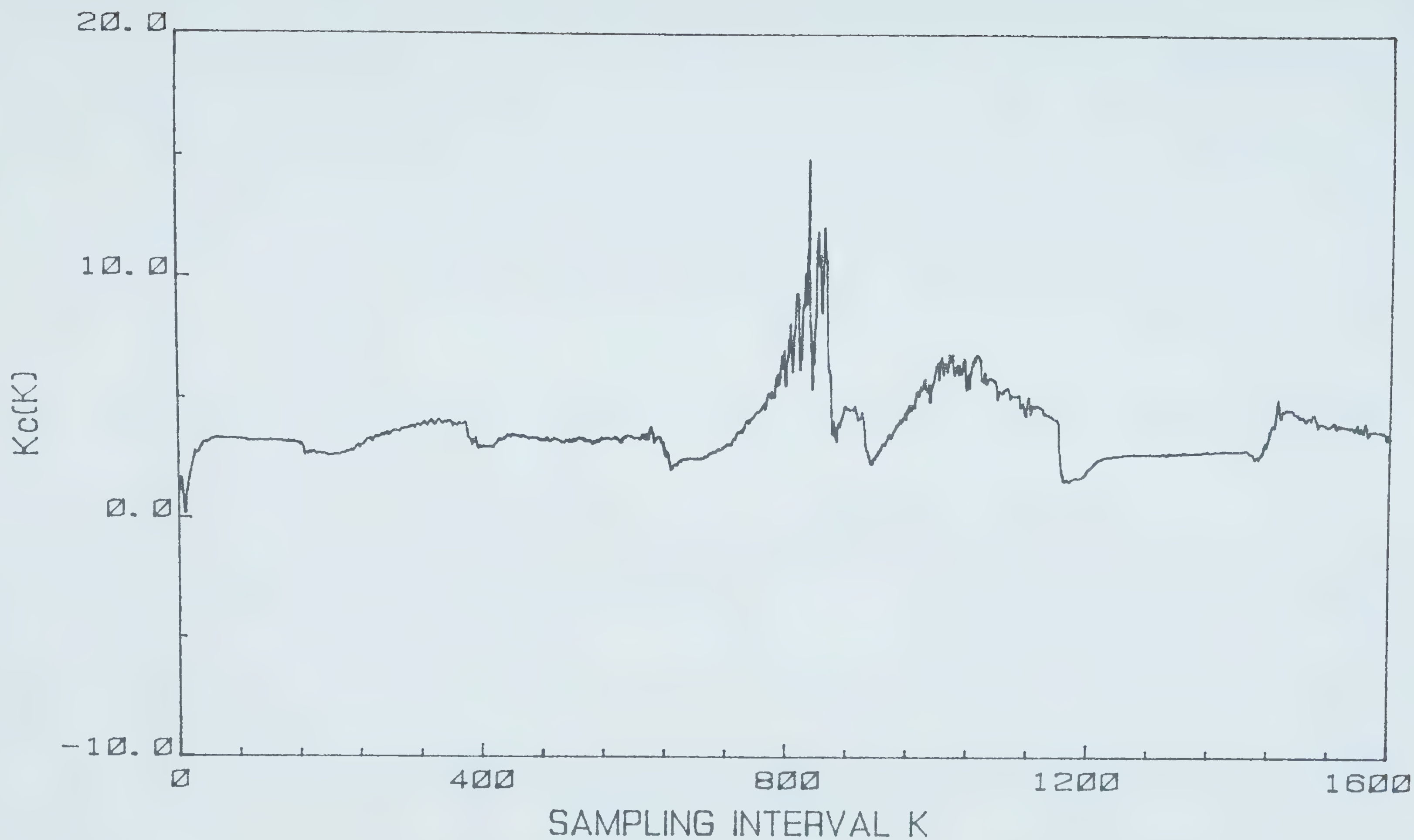

Figure 5.7a. Controller settings of adaptive PI controller
(Da=De=4/W=0.9/So=5)

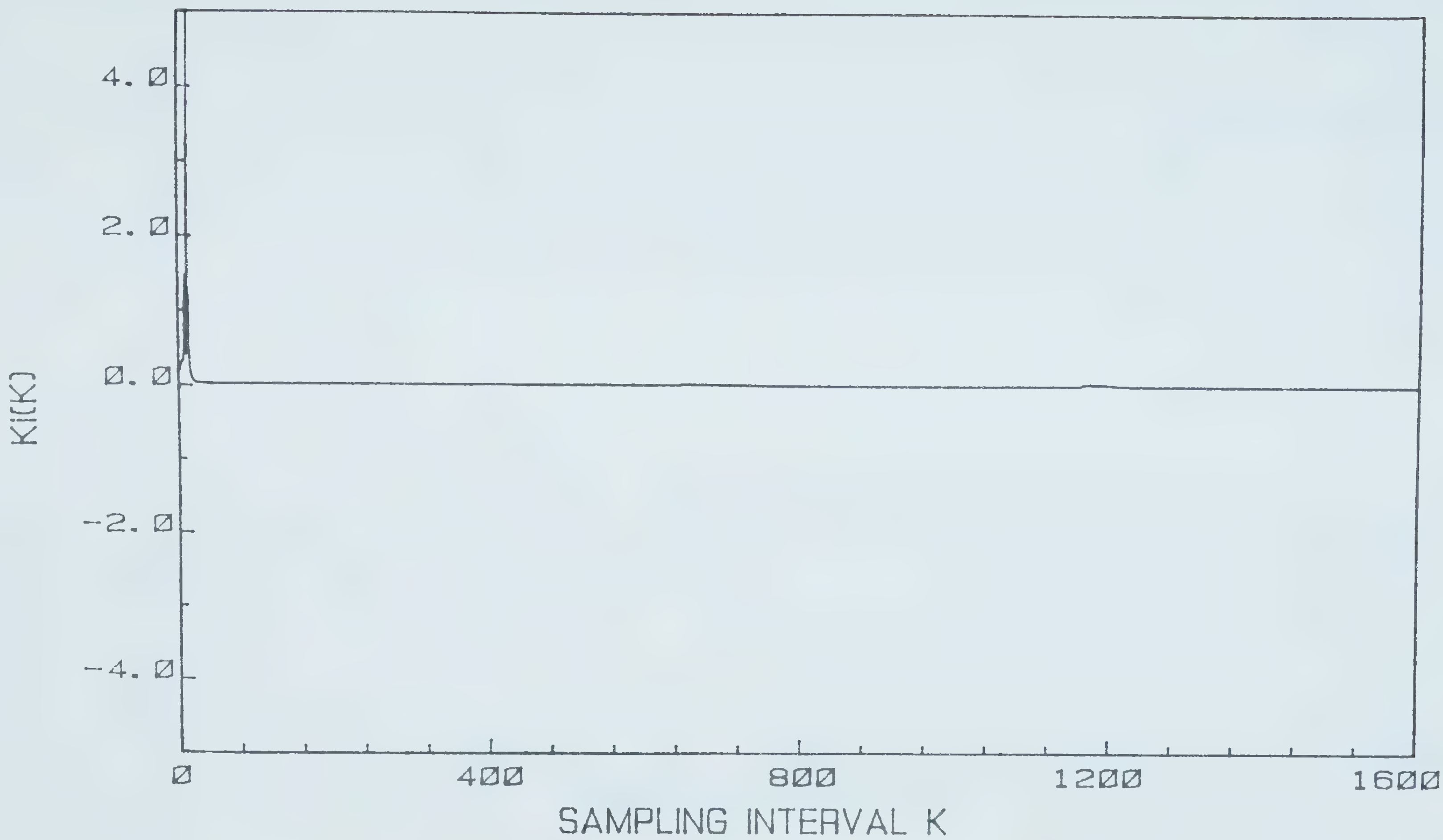

Figure 5.7b. Controller settings of adaptive PI controller (Da=De=4/W=0.9/So=5)

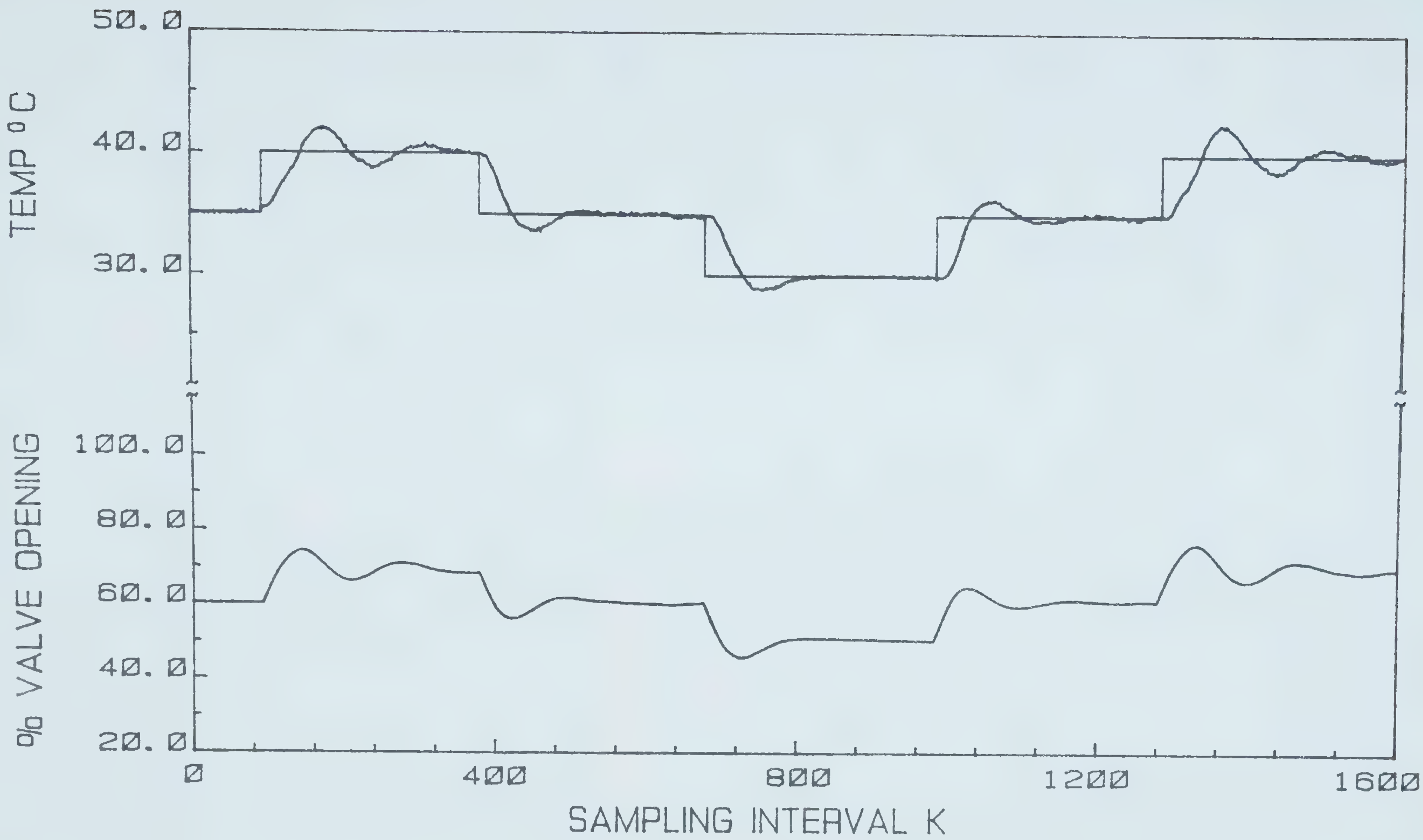

Figure 5.8. Stirred-tank heater response using fixed gain PID
(Da=4/Kc=5/Ki=0.005/Td=10)

transient stage (steady-state matching is always easy to do). In addition, since the use of $\Sigma\hat{b}_i$ in place of $B(z^{-1})$ removes the delay, it follows that the control action is faster than it would have been with $B(z^{-1})$.

5.3.2 Constant but Unknown Time Delay

When the time delay is unknown, it is difficult to determine the number of extra $\hat{b}$ parameters required to be estimated. To evaluate the adaptive PID(PI) controller performance in such a situation, the time delay of the stirred-tank heater is assumed to be unknown. Although the actual time delay is 4 sampling periods with sampling interval of 4 seconds, the number of extra $\hat{b}$ parameters was intentionally specified to be different than 4. In this section, two different runs with two and six extra parameters are used.

Figure 5.9 and Figure 5.11 show the stirred-tank heater responses using adaptive PID controller when the number of extra $\hat{b}$ parameters are two and six respectively. Similarly, Figure 5.10 and Figure 5.12 show the stirred-tank heater responses using adaptive PI controller with the rest of the operating conditions remain unchanged. The large initial variations in the input-output responses are due to the uncertainty in the initial controller parameters. From equations (2.33), (2.37), (2.38) and (2.39), though the desired closed-loop poles are pre-selected, the PID controller constants will vary according to the parameter

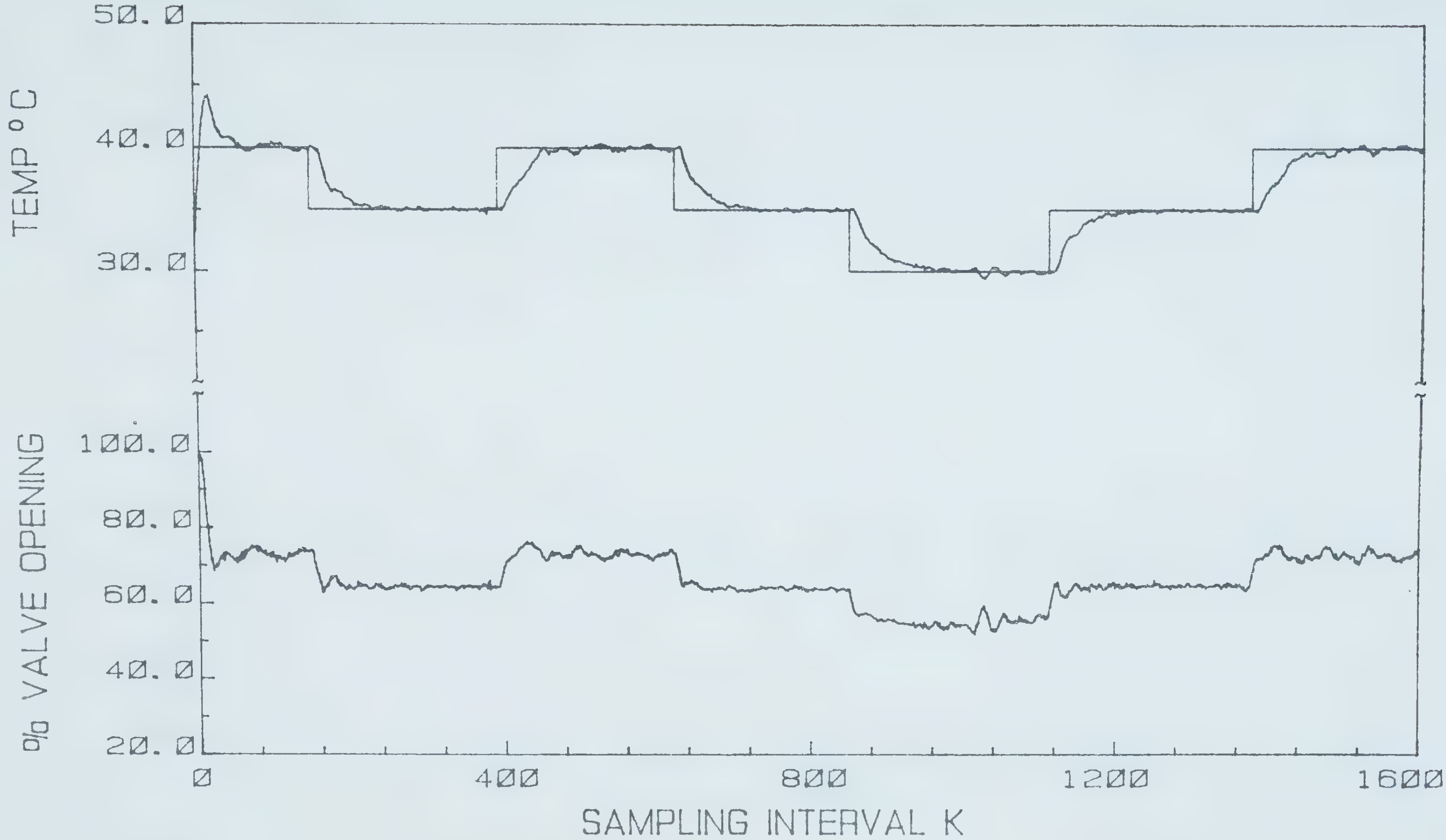

Figure 5.9. Stirred-tank heater response using adaptive PID controller (De=2/Da=4/W=0.9/So=5)

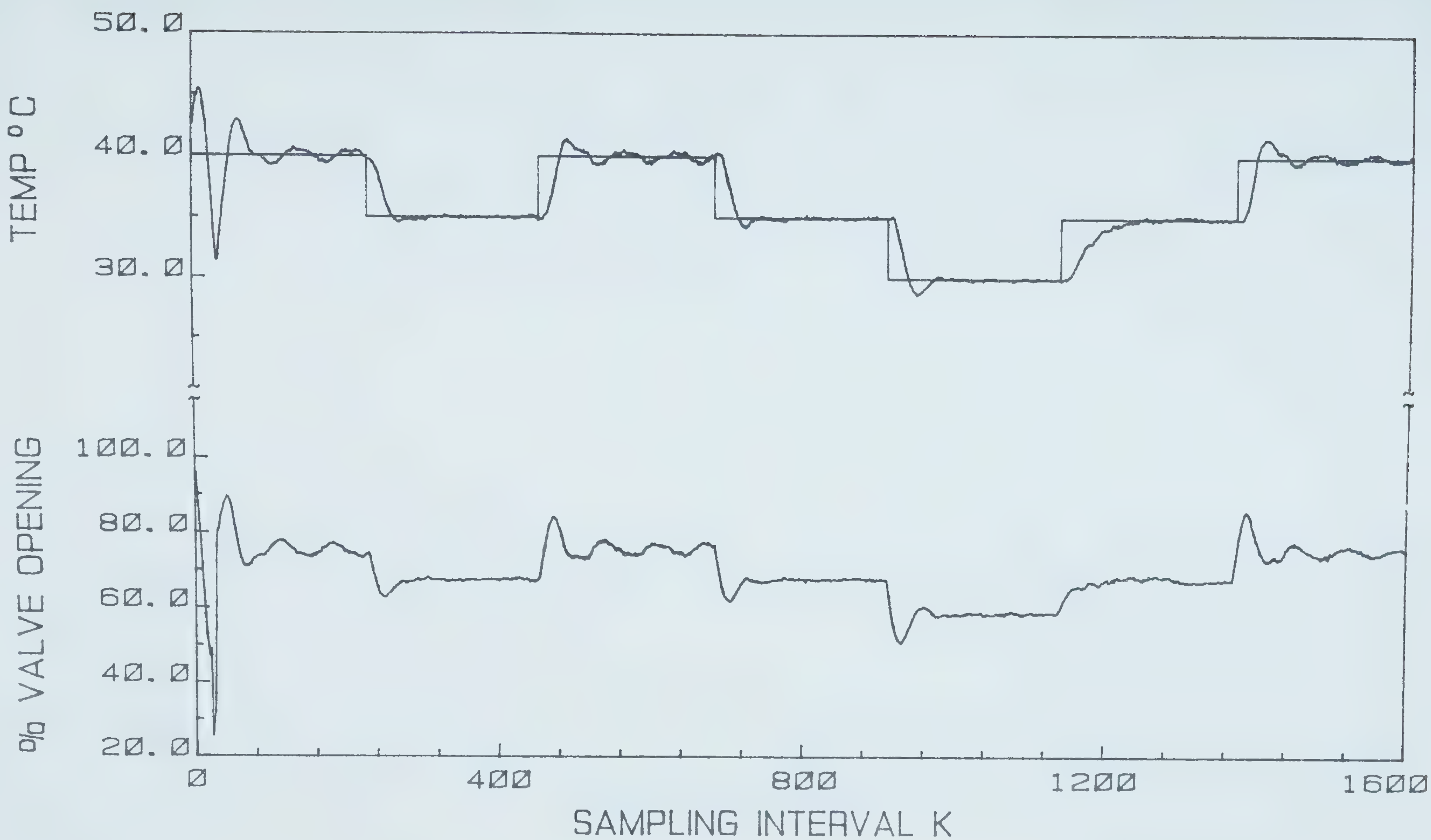

Figure 5.10. Stirred-tank heater response using adaptive PI controller (De=2/Da=4/W=0.9/So=5)

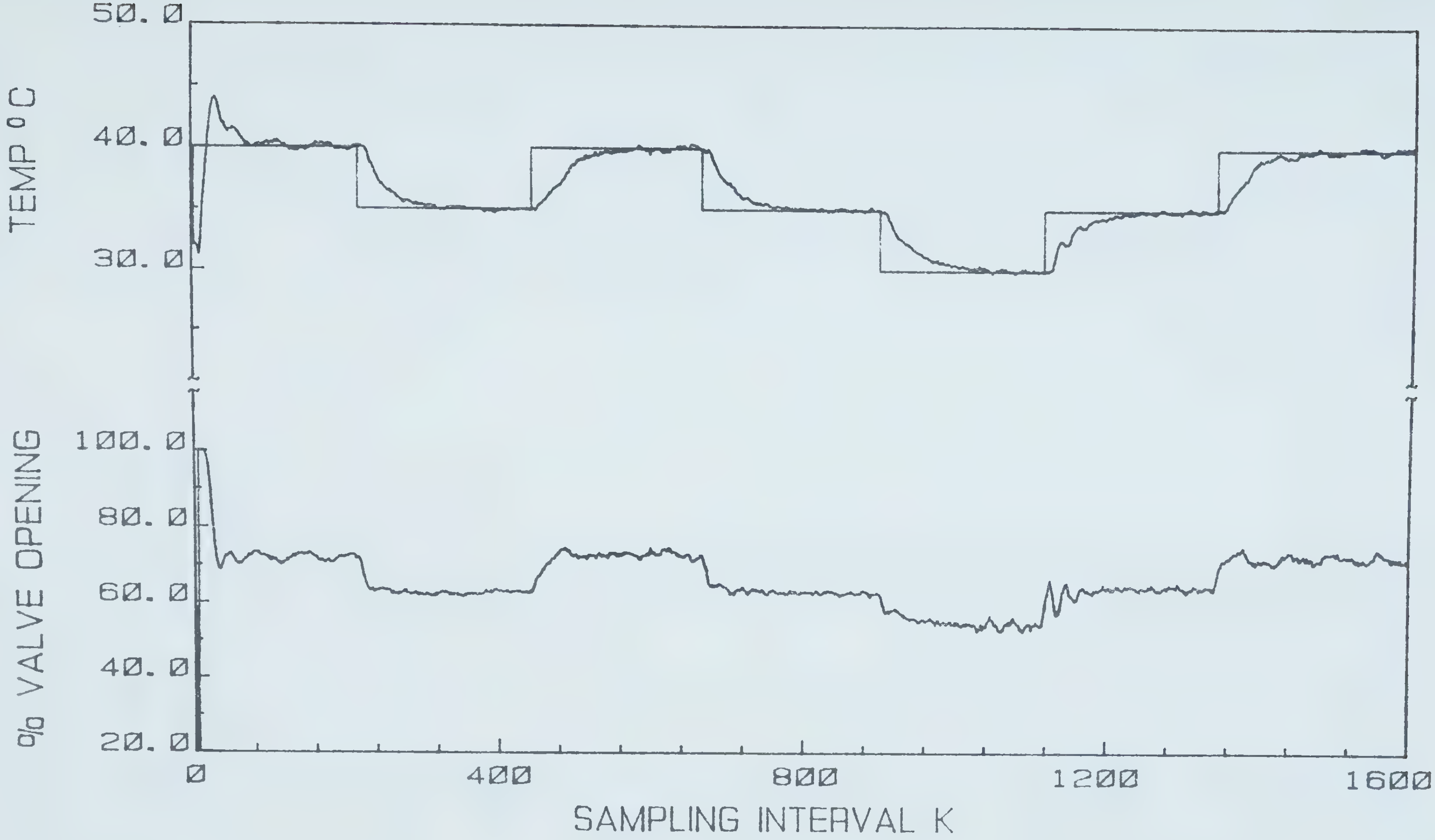

Figure 5.11. Stirred-tank heater response using adaptive PID controller (De=6/Da=4/W=0.9/So=5)

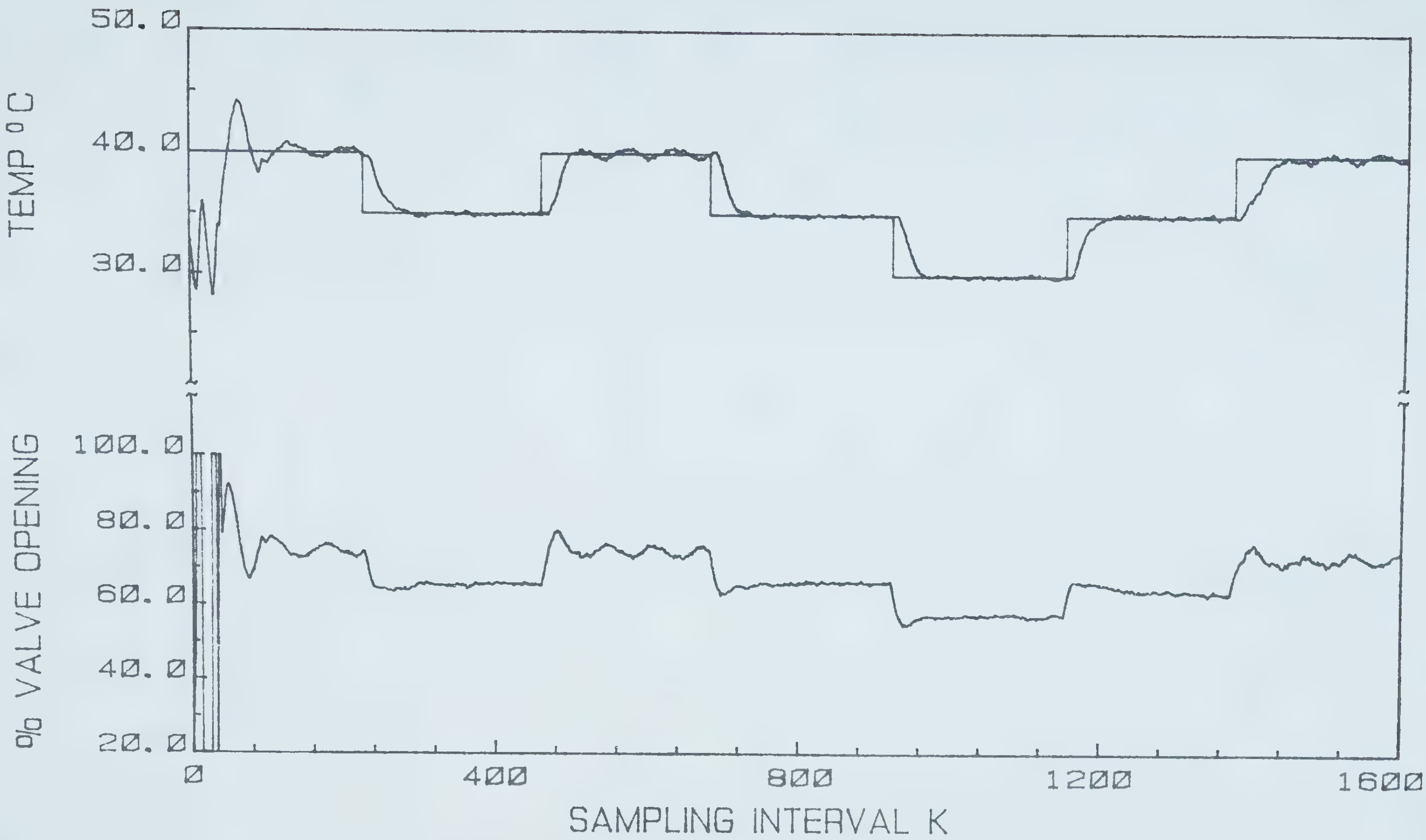

Figure 5.12. Stirred-tank heater response using adaptive PI controller (De=6/Da=4/W=0.9/So=5)

estimates. Therefore, it is expected to have these large variations during the initial period until parameter convergence occurs. At the same time, it is also clear that a parameter estimation algorithm with fast convergence property is the most desirable. Though there are more variations in the input and output responses during the initial period for the adaptive PI controller, all responses show excellent performance of the adaptive PID(PI) controller regardless of the number of extra $\hat{b}$ parameters. This property of the adaptive PID(PI) controller is very significant, in particular for systems with unknown and/or changing time delay dynamics. Comparison of the adaptive PID and PI control in Figure 5.9 to Figure 5.12, however, shows the better performance by the adaptive PID control. This is probably because of the presence of time delay and also the assumption of a lower order model for PI control, i.e. a first order model.

5.3.3 Unknown and/or Changing Time Delay

The time delay of the stirred-tank heater can be varied by using different thermocouples to measure the outlet temperature. When the sampling interval is 4 seconds, the system time delay varies from one sampling period at thermocouple B to two sampling periods at thermocouple C and four sampling periods at thermocouple E. In this section, the number of extra $\hat{b}$ parameters are chosen to be three. The reason for this choice is again to illustrate the

performance of the adaptive PID(PI) controller in the presence of time delay mismatch. More specifically, it assumes that system time delay is unknown.

Figure 5.13 shows the response of the stirred-tank heater just after being switched from the fixed gain PI controller to the adaptive PID controller. The time delay during the first three setpoints is four sampling periods. At the fourth setpoint change, the time delay is changed to two sampling periods by switching thermocouples. Finally, the time delay is changed to one sampling period at the sixth setpoint change. When three extra $\hat{b}$ parameters are used in the estimation model, it is theoretically indicating to the estimator that there is a delay of three sampling periods in the system. Ideally, the three leading $\hat{b}$ parameters will be zero or close to zero for a delay of three sampling periods. In the presence of noise, however, it is difficult to distinguish between zero and non-zero parameters. In addition, the actual time delay in this case is changing from four to two and one sampling period. This error in the delay model makes the differentiation between zero and non-zero parameters even harder. Giving a sequence of input and output measurements, the estimator will optimize the pre-specified model parameters, i.e. minimize the estimation error. Since only the value of $\Sigma\hat{b}_i$ is used in the control law calculation, the problem of distinguishing between zero and non-zero parameters is thus avoided. Despite the slow response at the first setpoint change,

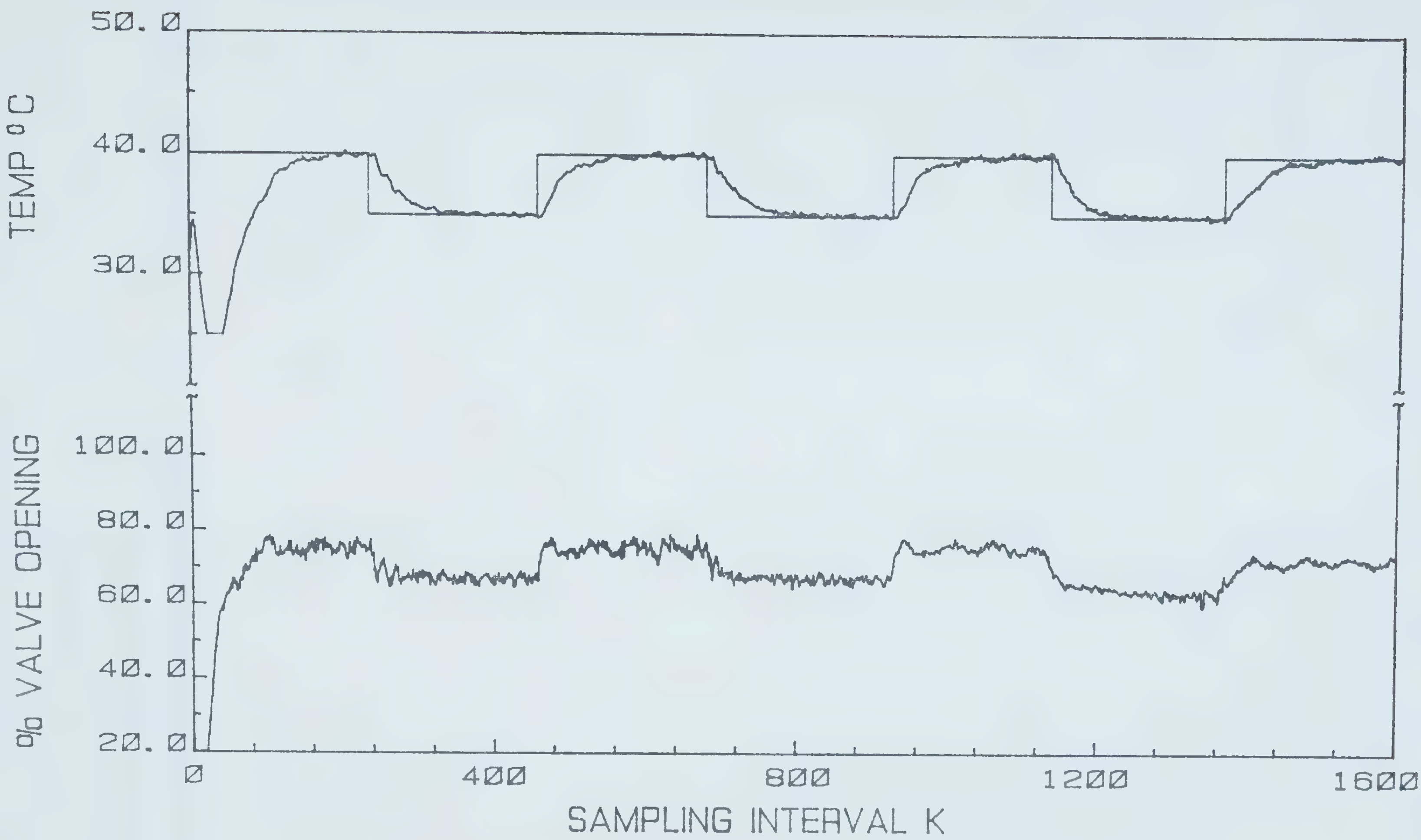

Figure 5.13. Stirred-tank heater response using adaptive PID controller (De=3/Da=4,2,1/W=0.9/So=5)

excellent performance is obtained at the subsequent setpoint changes. The corresponding adaptive PID controller settings are shown in Figure 5.14a, 5.14b and 5.14c; the corresponding forgetting factor is plotted in Figure 5.15.

At the instant at which setpoint changes are introduced, there are small variations occur in Kc. These variations are more obvious when the setpoint change is accompanied by a change in the time delay, e.g. at k=690. Larger variations are also encountered when the time delay is changed from 2 to 1 sampling period, e.g. at k=1350. These initial variations in Kc are due to the mismatch in the time delay model, in particular the $\hat{b}$ parameters. For convinience, equations (2.37), (2.38) and (2.39) are rearranged here as:

$$Kc = \frac{\Sigma w_i}{w_1 \; \Sigma \hat{b}_i} (\hat{a}_1 + 2\hat{a}_2) \tag{5.2}$$

$$T_d = - \frac{\hat{a}_2 \; T_s}{\hat{a}_1 + 2\hat{a}_2} \tag{5.3}$$

$$T_i = - \frac{T_s \; (\hat{a}_1 + 2\hat{a}_2)}{1 + \hat{a}_1 + \hat{a}_2} \tag{5.4}$$

When time delay varies, the process dynamics change and so do the parameter estimates. Since the sum of $\hat{b}$ parameters appears in the denominator of equation (5.2), any changes in this sum will lead to significant variations in Kc. As shown

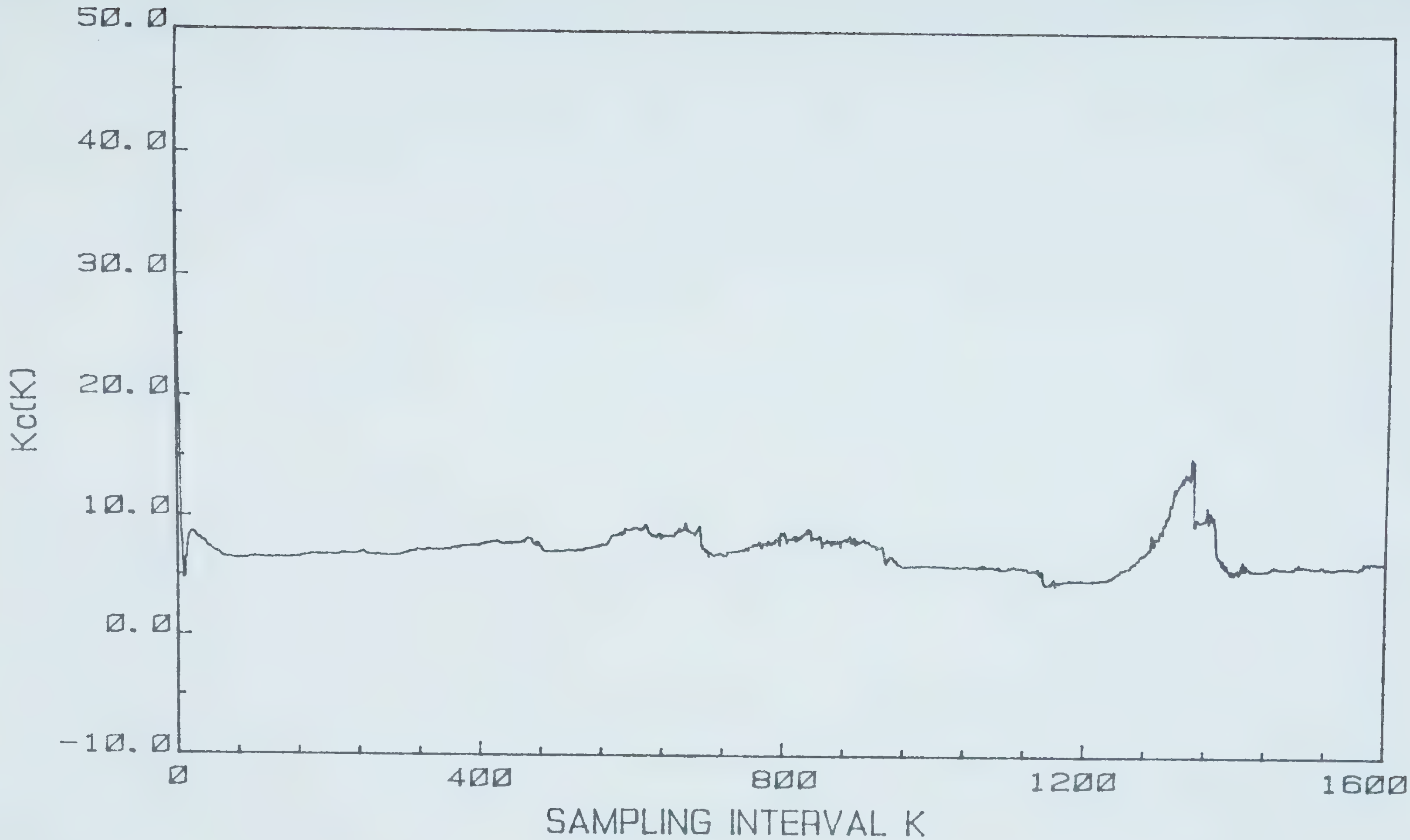

Figure 5.14a. Controller settings of adaptive PID controller (De=3/Da=4,2,1/W=0.9/So=5)

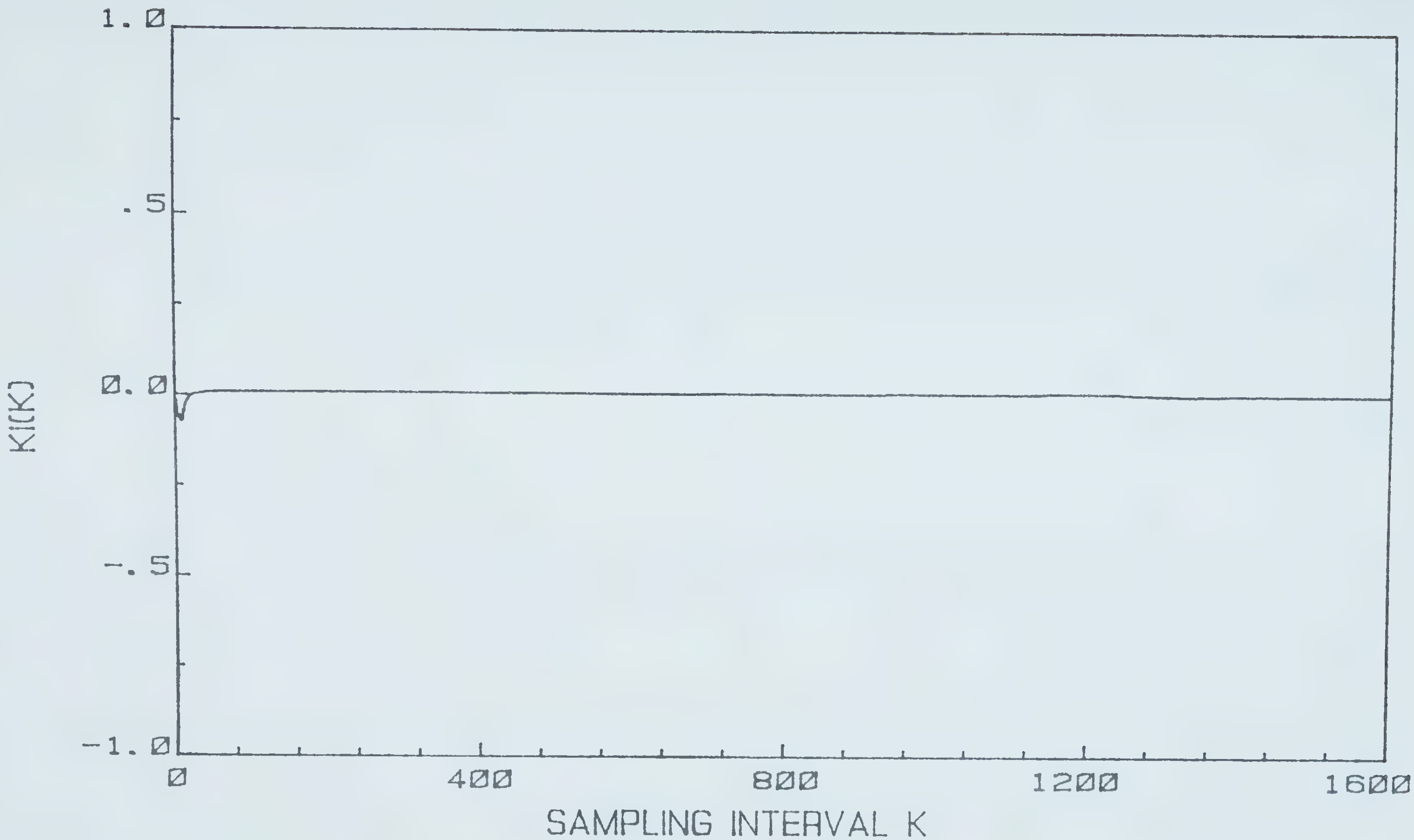

Figure 5.14b. Controller settings of adaptive PID controller (De=3/Da=4,2,1/W=0.9/So=5)

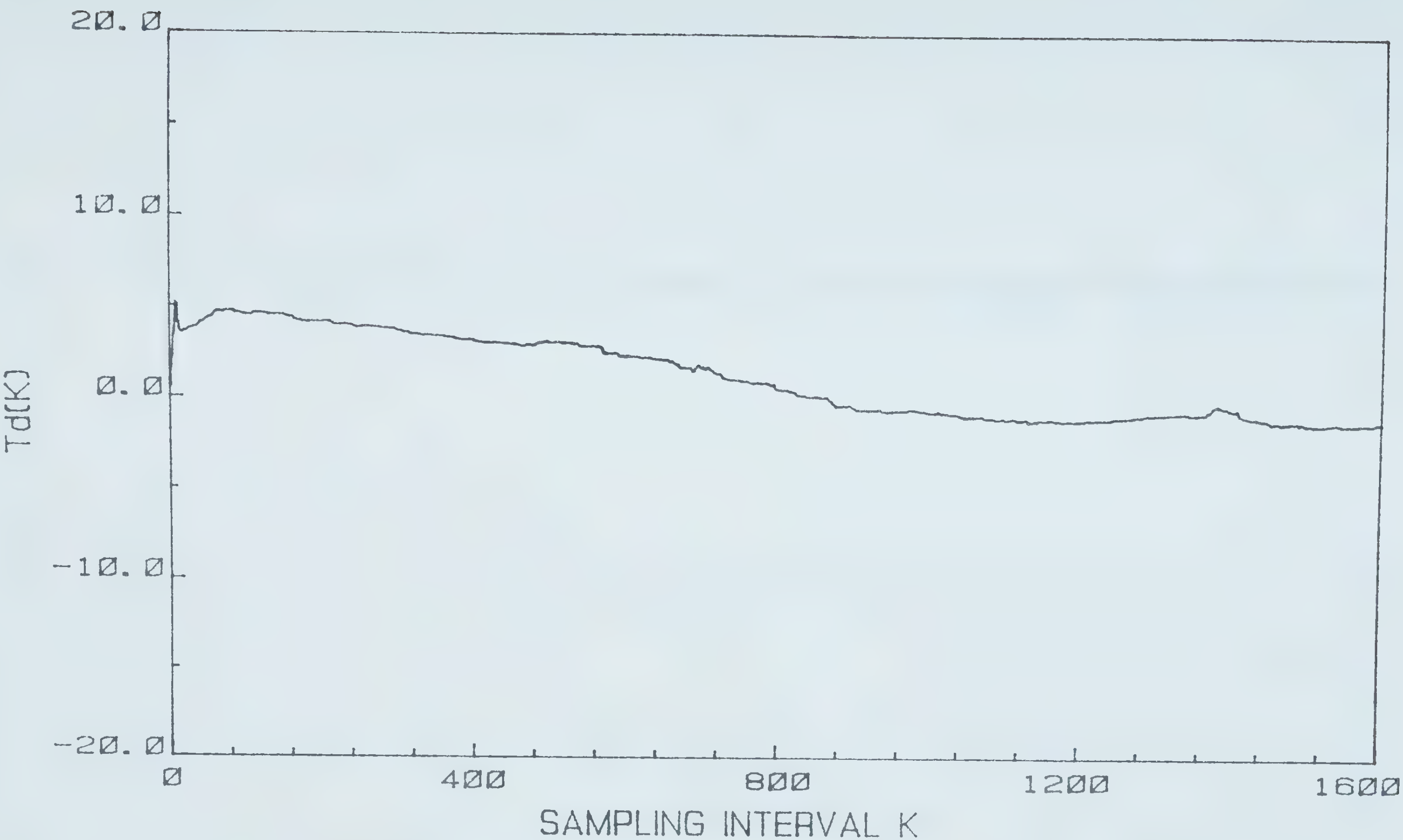

Figure 5.14c. Controller settings of adaptive PID controller (De=3/Da=4,2,1/W=0.9/So=5)

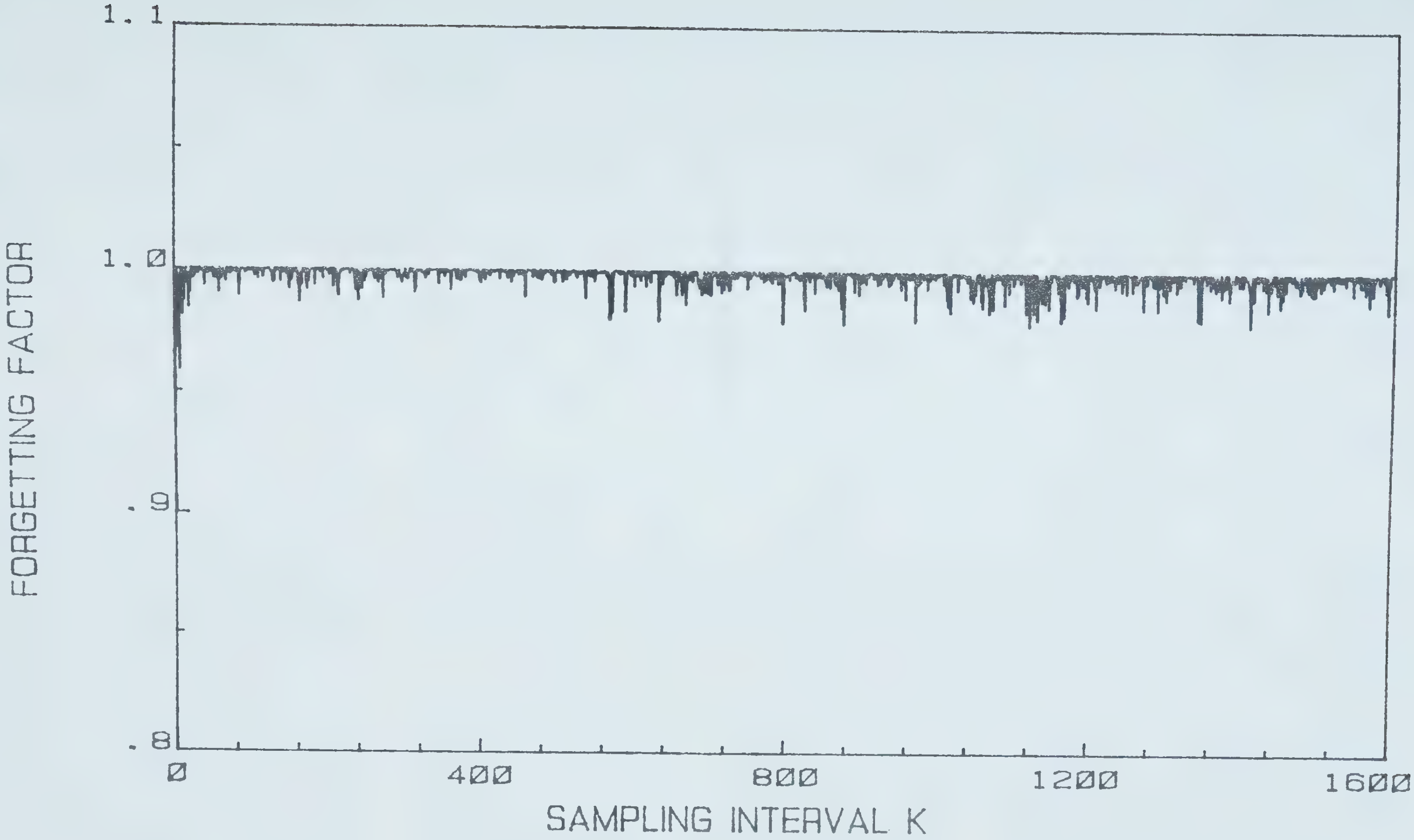

Figure 5.15. Forgetting factor of adaptive PID controller (De=3/Da=4,2,1/W=0.9/So=5)

in equations (5.3) and (5.4), the values of T_d and T_i are effected only by the sample time and the $\hat{a}$ parameters. However, it should not be assumed that the values of T_d and T_i are independent of time delay. This is because the $\hat{a}$ parameters also vary with time delay (see Table 4.2).

Figure 5.16 shows the stirred-tank heater response controlled by an adaptive PI controller, and Figure 5.17 shows the stirred-tank heater response under the control of a fixed gain PID controller. Variations in the stirred-tank heater response are larger with the fixed gain PID controller. It also takes a longer period of time to reach steady-state whenever a setpoint change is made. For this reason, Figure 5.17 shows only 5 setponit changes eventhough the total number of setpoint changes are essentially the same as the rest of the runs for unknown and/or varying time delay case. In Figure 5.17, the initial time delay is estimated to be four sampling periods. This time delay is changed to two sampling periods at the second setpoint change, and then changed to one sampling period at the fourth setpoint change.

The system time delay is also varied in the reversed direction, i.e. from one sampling period at thermocouple B to four sampling periods at thermocouple E. Figures 5.18, 5.19 and 5.20 show the corresponding stirred-tank heater responses with an adaptive PID controller, an adaptive PI controller and a fixed gain PID controller respectively. The same observations as mentioned above apply with respect to

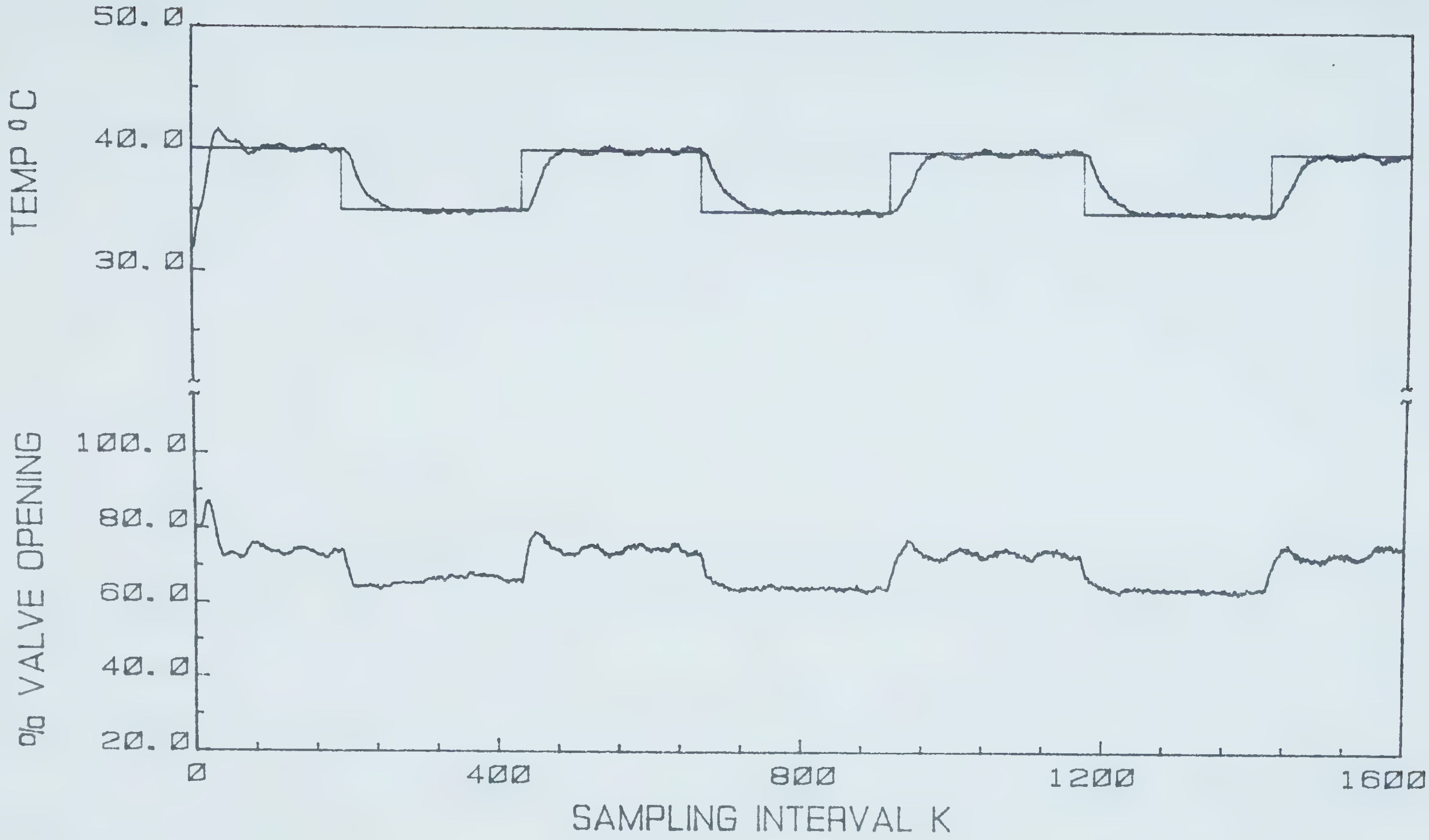

Figure 5.16. Stirred-tank heater response using adaptive PI controller (De=3/Da=4,2,1/W=0.9/So=10)

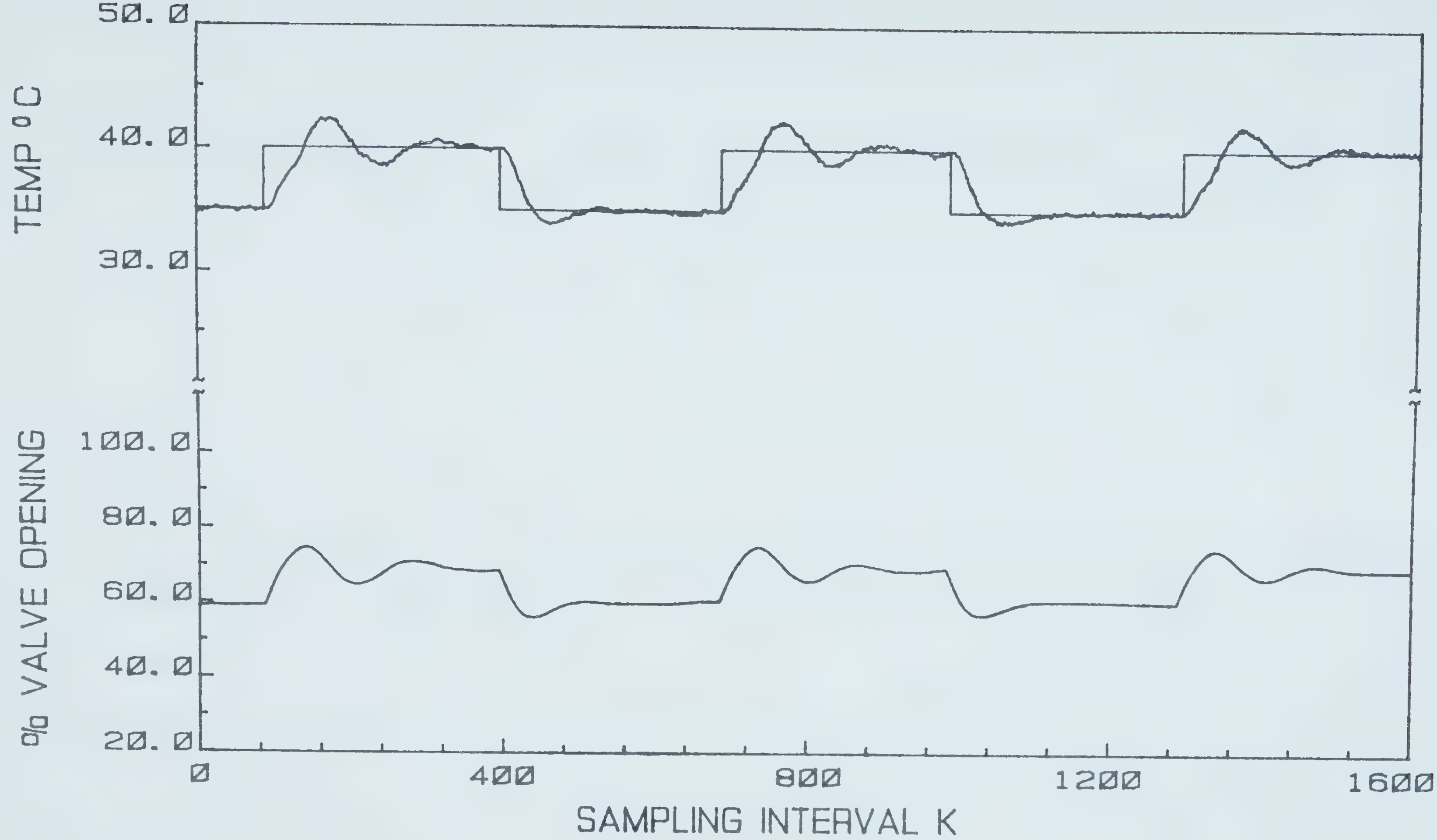

Figure 5.17. Stirred-tank heater response using fixed gain PID
(Da=4,2,1/Kc=5/Ki=0.005/Td=10)

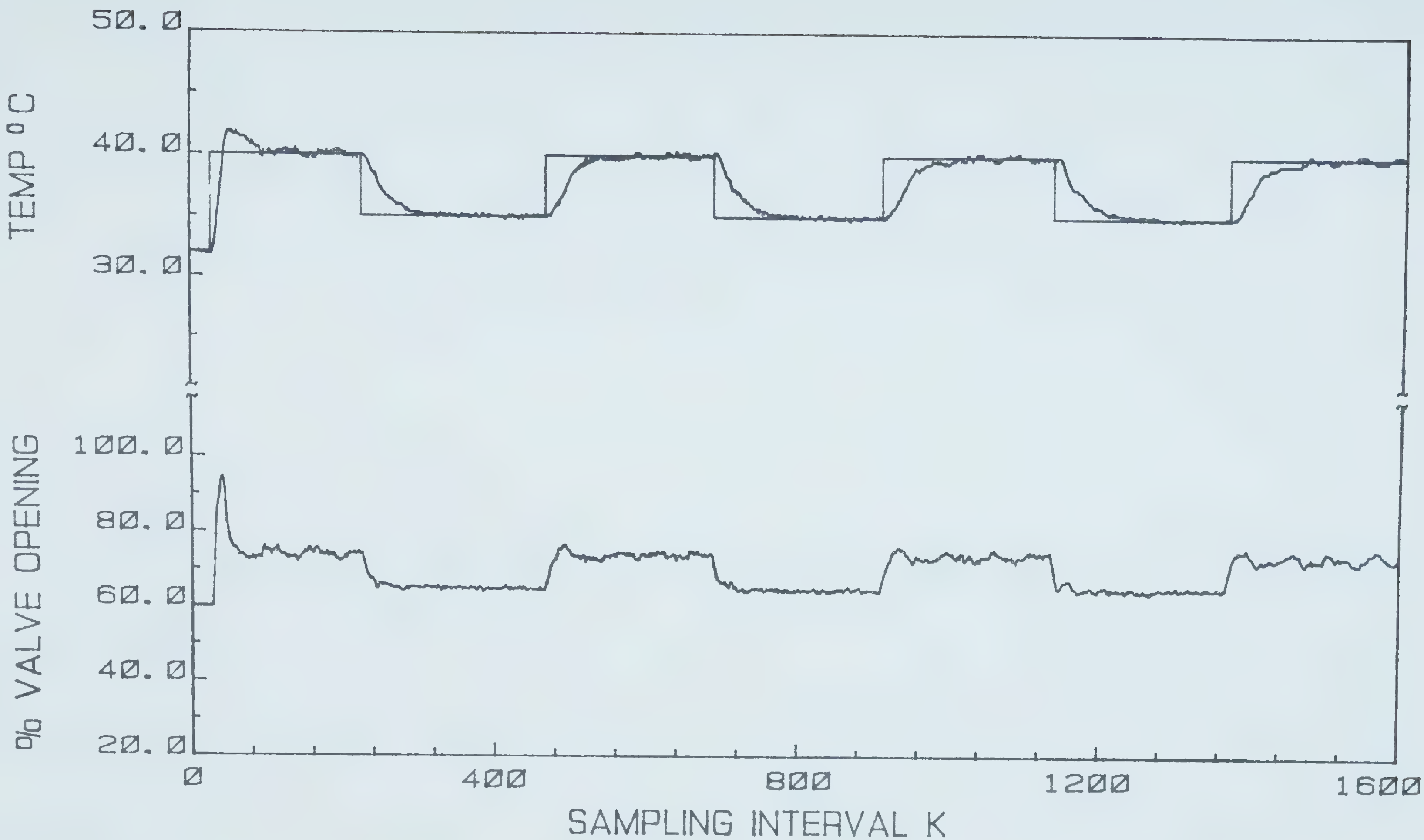

Figure 5.18. Stirred-tank heater response using adaptive PID controller (De=3/Da=1,2,4/W=0.9/So=10)

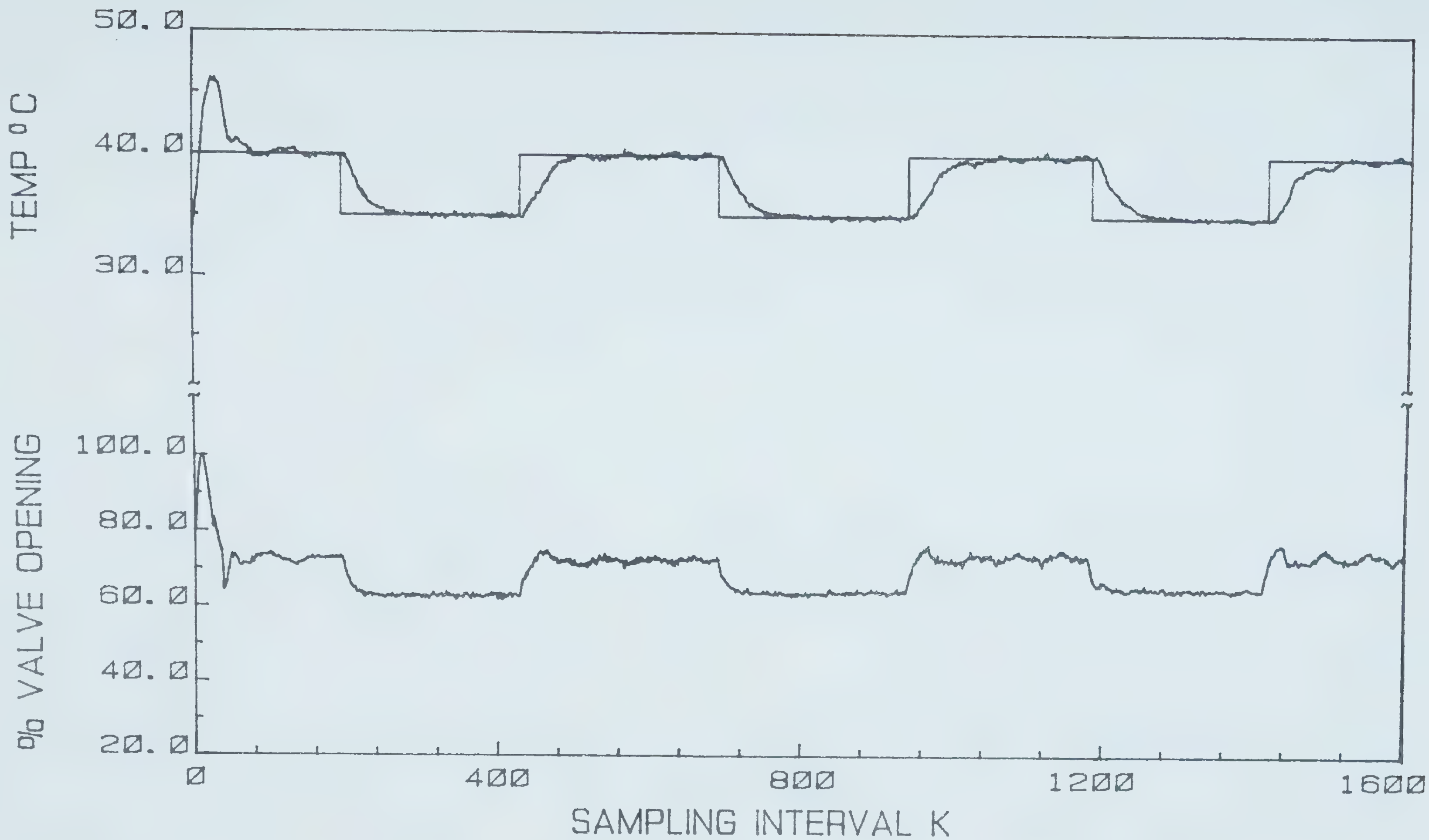

Figure 5.19. Stirred-tank heater response using adaptive PI controller (De=3/Da=1,2,4/W=0.9/So=10)

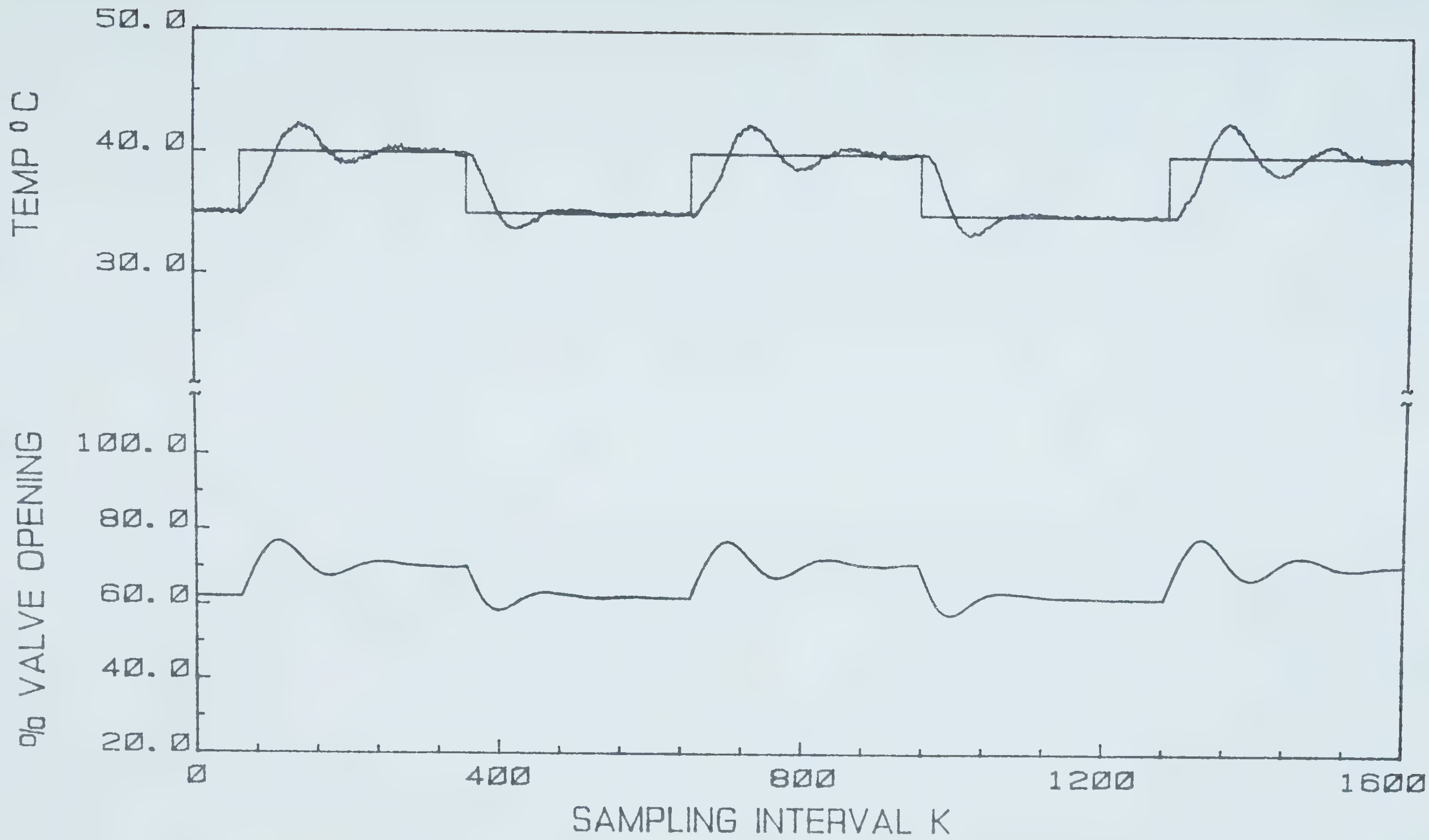

Figure 5.20. Stirred-tank heater response using fixed gain PID (Da=1,2,4/Kc=5/Ki=0.005/Td=10)

the performance of fixed gain PID controller. If frequent setpoint changes are required, the best performance is obtained with the adaptive PID or PI controllers.

One possible drawback of adaptive controllers, in general, is that variations in the controller output are larger than those of the fixed gain controller. Self-tuning regulators, for example, have characteristics that result in excessive control action. This leads to the introduction of a weighting function on the control action in the performance index, resulting in the so-called 'generalized' self-tuning controller. Pole-placement controllers, on the other hand, limit the control action by placing the closed-loop poles in appropriate locations. However, as found in this experimental study, the adaptive PID(PI) controller still gives slightly noisier control input than the fixed gain PID controller even when the response has reached steady state. This can be explained as follows. The adaptive PID(PI) controller takes the form of equation (2.25) where the present and past output measurements and setpoint are multiplied by Kc. Any small changes in Kc due to slight variations in the parameter estimates will give rise to slight variations in the control input. Many different techniques can be used to eliminate the small variations in the control input. One commonly used method is to filter the control input, e.g. by using an exponential filter or a velocity limiting filter. Another possible way is to filter the parameter estimates. The filtered

parameters can then be used to calculate the PID controller settings, however the unfiltered parameters would still be used for the next estimation. Since the PID controller settings are determined from the parameter estimates, it is also possible to filter the controller settings directly instead of the parameter estimates. Other methods to reduce the noise in the control input can also be found in the literature, e.g. Goodwin and Sin [1984].

5.3.4 Disturbance Rejection

At steady-state operation a mass flowrate of the inlet cold water of approximately 8.2 kg/min is maintained. To evaluate the adaptive PID controller performance in the presence of load disturbances, step load disturbances are introduced into the system by varying the inlet flowrate by ±1 kg/min.

Figure 5.21 shows the stirred-tank heater response to step disturbances in the inlet flowrate by using an adaptive PID controller. The corresponding step changes in the inlet flowrate is shown in Figure 5.22. At the point of switching from fixed gain PI controller to the adaptive PID controller, i.e. at the first step change in setpoint as shown in Figure 5.21, the large uncertainty in the estimated parameters results in the lowest possible forgetting factor value (Figure 5.24). Consequently, large variations occur in the controller settings during this initial tuning period. The controller settings are given in Figure 5.23a, 5.23b and

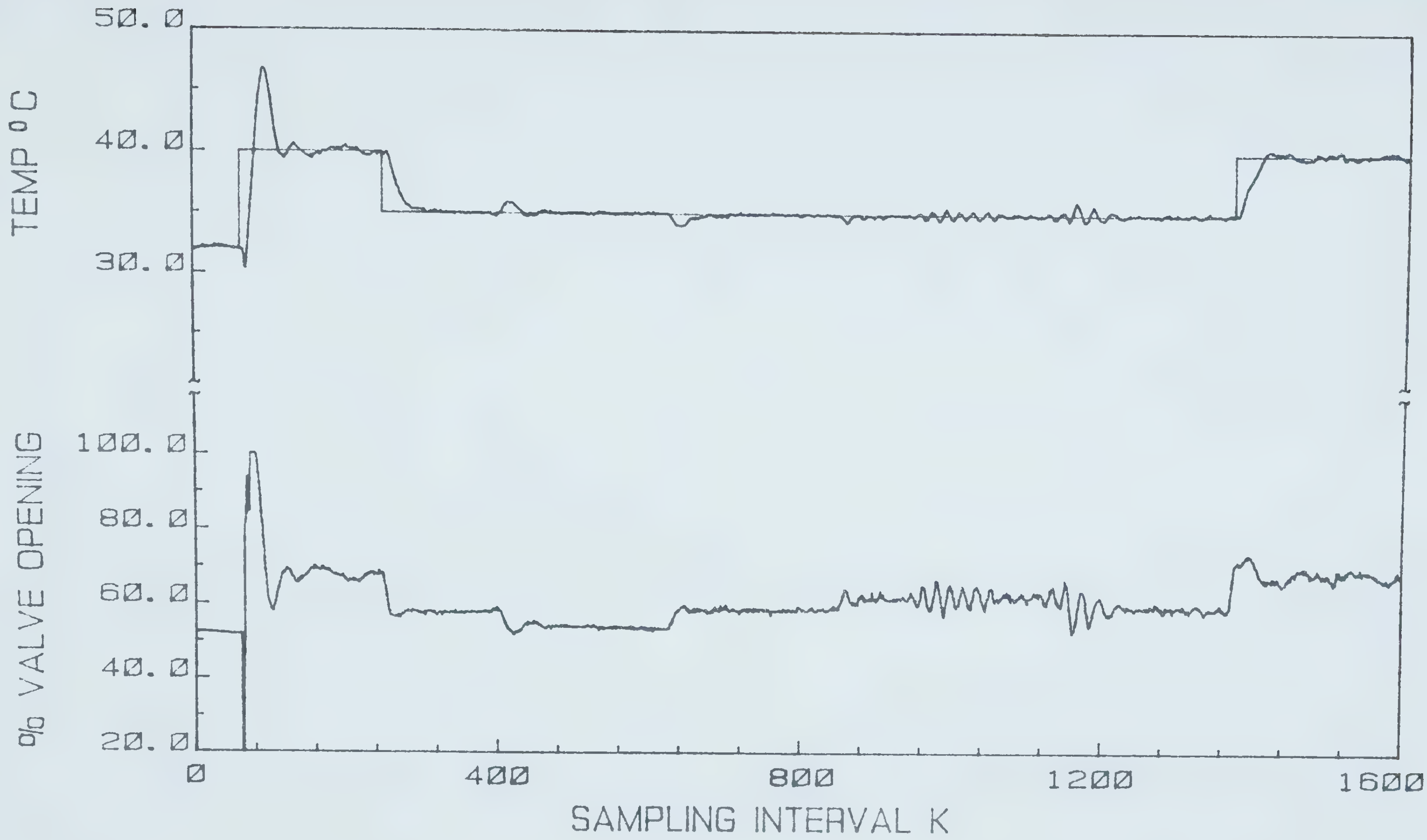

Figure 5.21. Stirred-tank heater response to step disturbances using adaptive PID controller w.o.ffd. (De=3/Da=4/W=0.9/So=10)

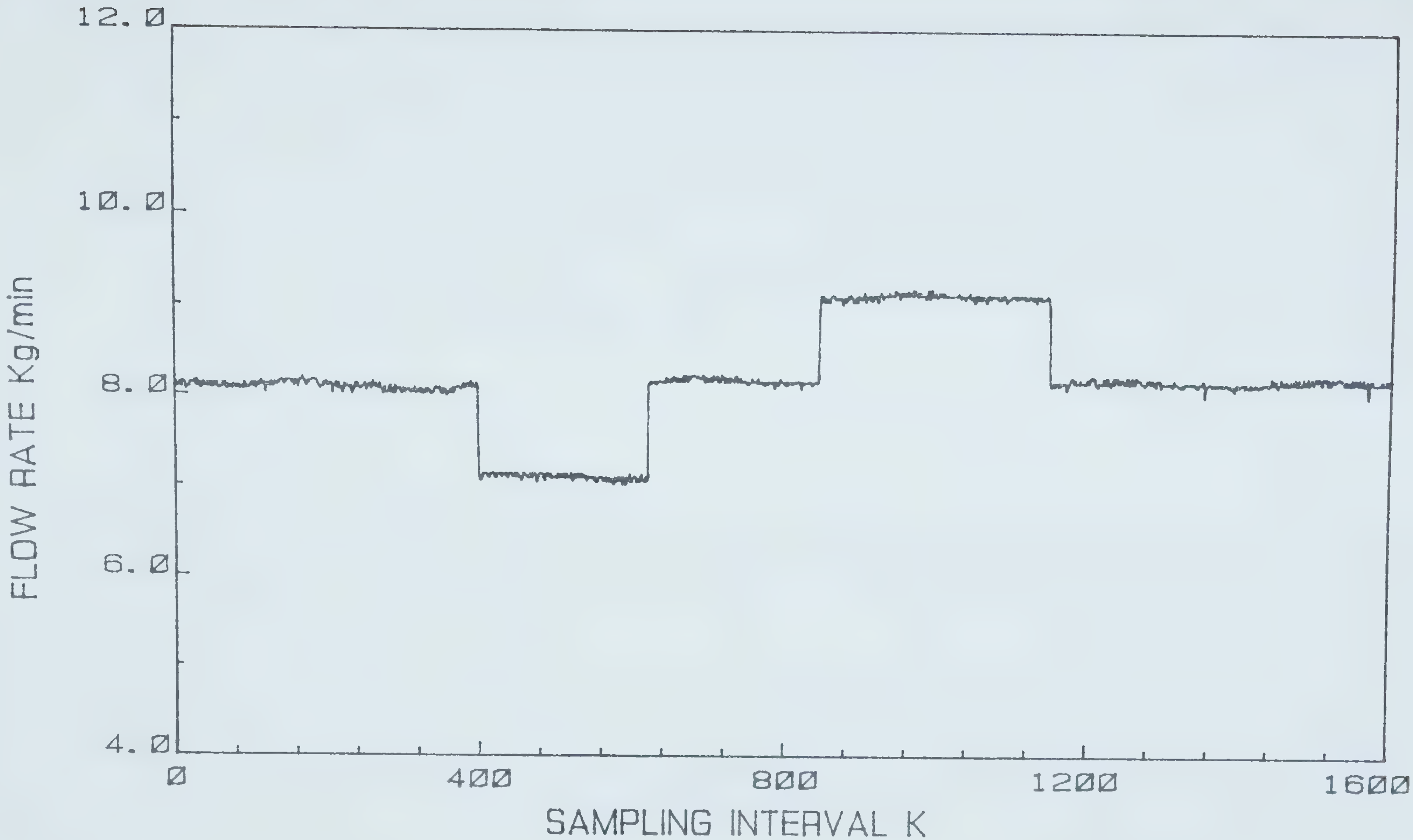

Figure 5.22. Step disturbances introduced to the stirred-tank heater using adaptive PID controller w.o.ffd. (De=3/Da=4/W=0.9/So=10)

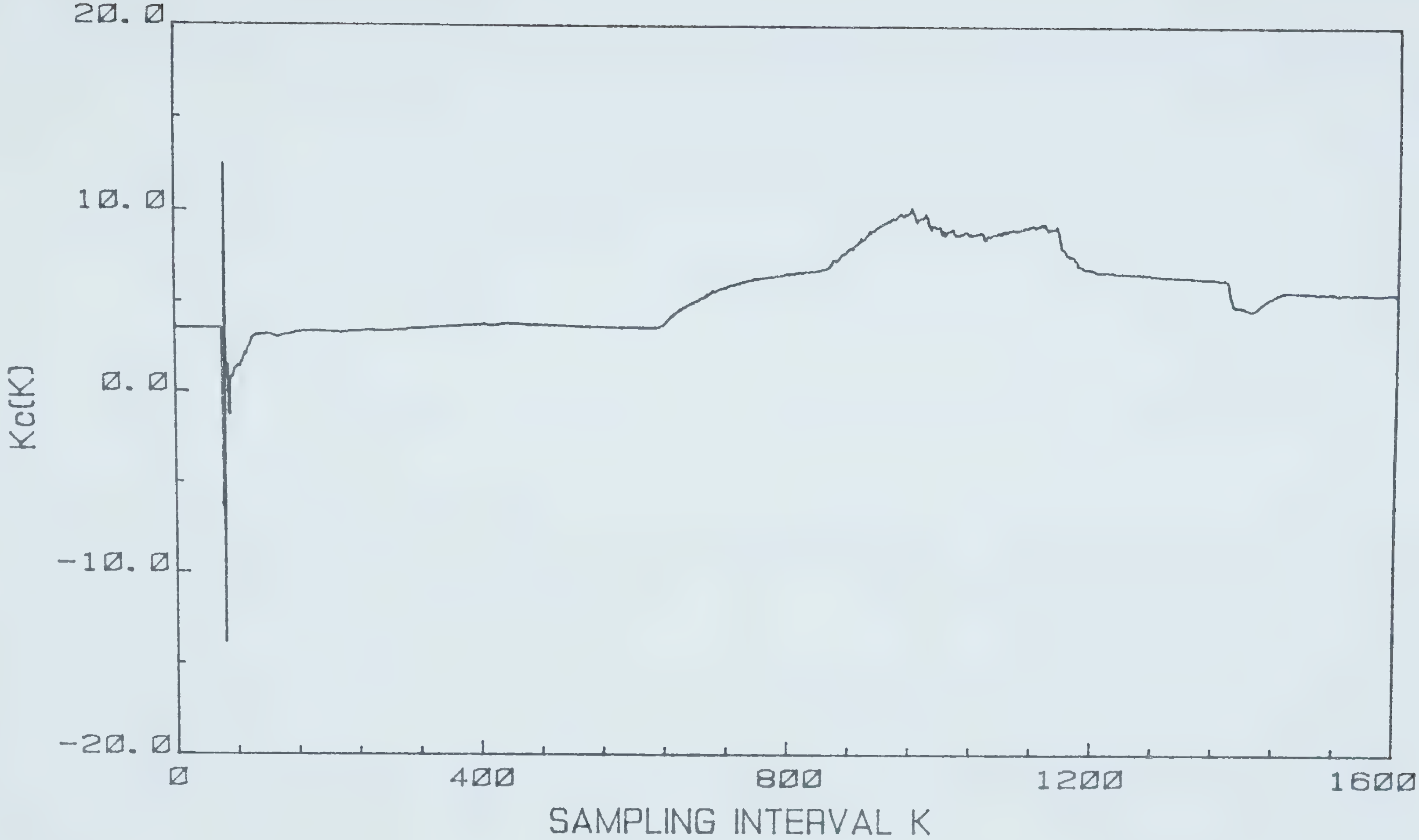

Figure 5.23a. Controller settings of adaptive PID controller (De=3/Da=4/W=0.9/So=10)

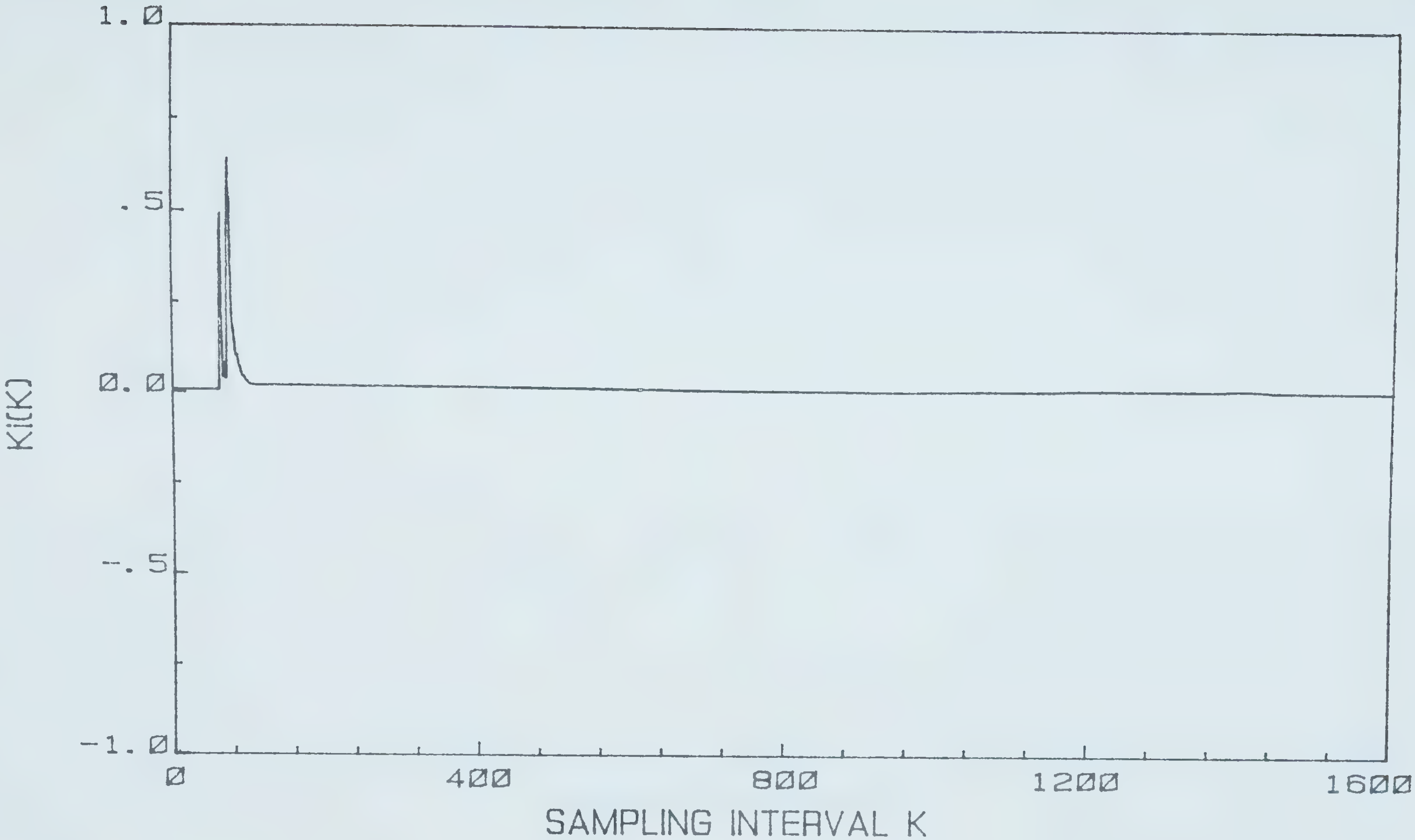

Figure 5.23b. Controller settings of adaptive PID controller (De=3/Da=4/W=0.9/So=10)

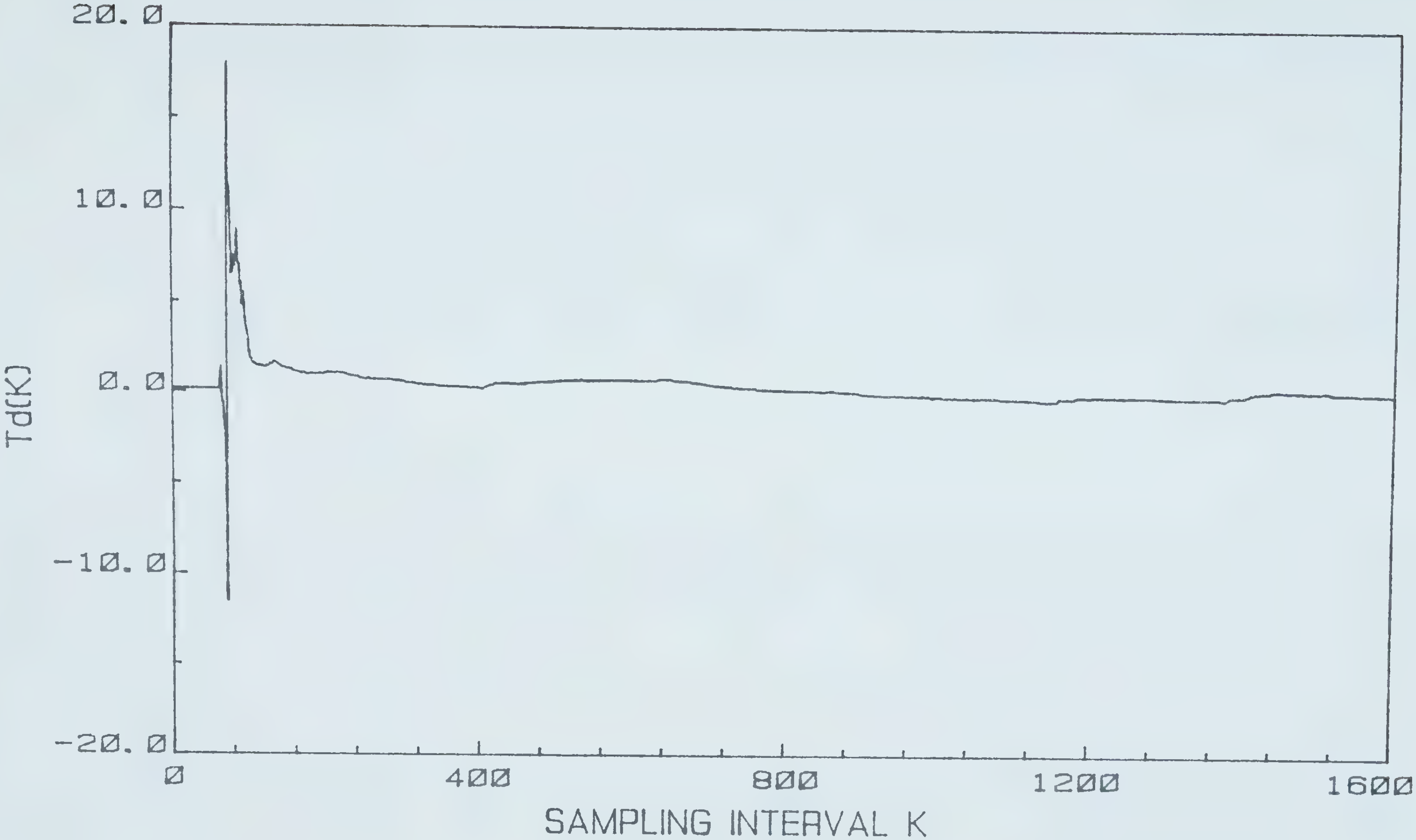

Figure 5.23c. Controller settings of adaptive PID controller
(De=3/Da=4/W=0.9/So=10)

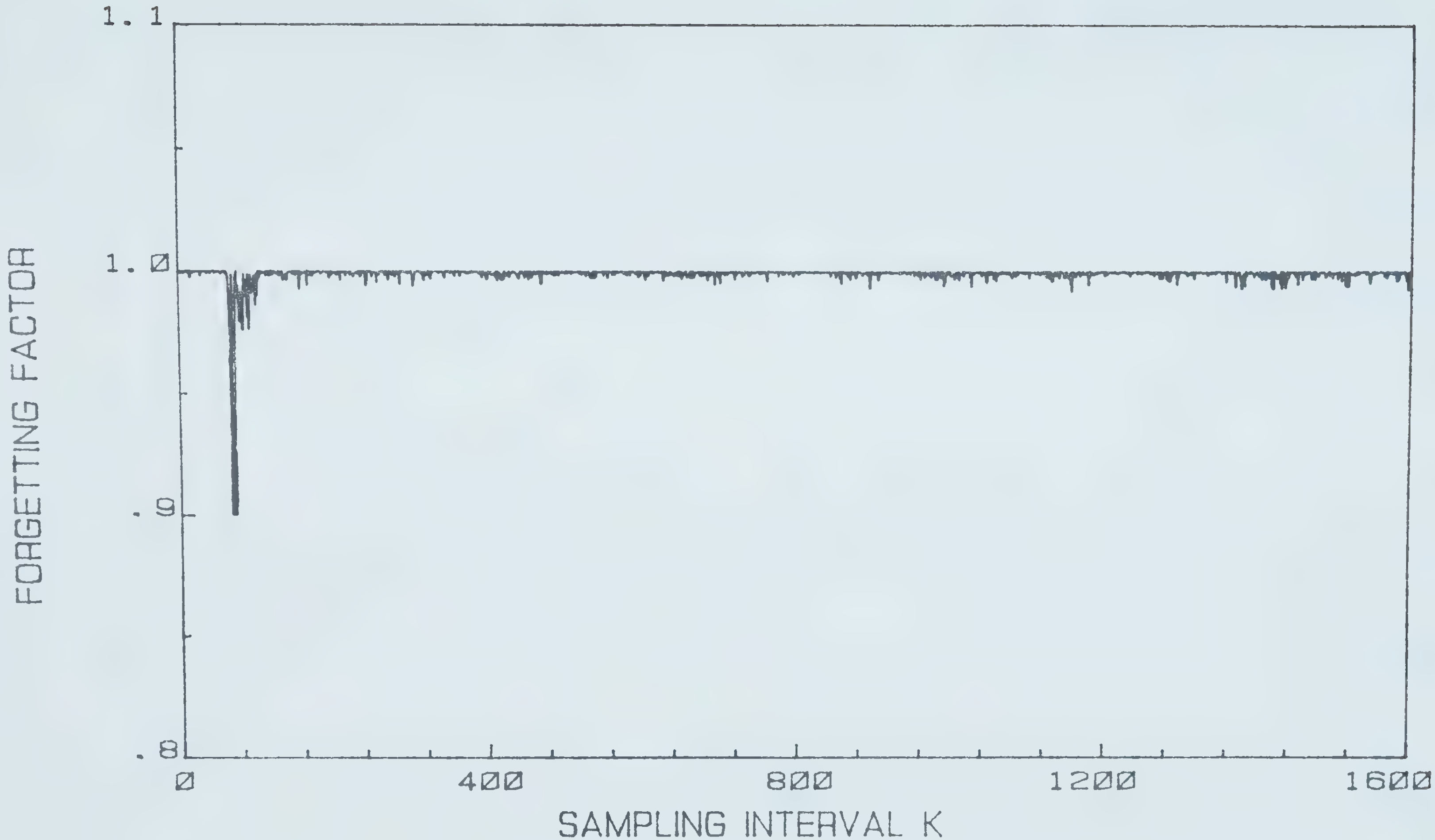

Figure 5.24. Forgetting factor of adaptive PID controller
(De=3/Da=4/W=0.9/So=10)

5.23c.

From the plot of the controller settings, the controller gain Kc increases to a higher value after the disturbances are introduced. This is due to the fact that the controller is first tuned to track setpoint changes. Theoretically, the controller gain is higher when tuning for regulatory purposes, for example open-loop methods with IAE or ITAE tuning criteria. For the stirred-tank heater, however, a negative step change in inlet flowrate does not increase the controller gain as much as a positive step change. This can probably be explained as follows. Due to the small heating surface and the open-top tank, it takes a longer period of time to heat up the fluid inside the tank than to cool it down, i.e. a non-linear system. When the inlet flowrate is increased, the temperature drops. If the controller gain were to remain constant, it would take a longer period of time to heat the fluid temperature up to the steady-state level. Figure 5.26 shows the stirred-tank heater response with a fixed gain PID controller and Figure 5.27 shows the corresponding changes in the inlet flowrate. Comparison of Figure 5.26 to Figure 5.21 shows the slower response of the fixed gain PID controller with respect to both regulatory and servo control. A similar test was also carried out on an adaptive PI controller, and the result is shown in Figure 5.25. The better performance by the adaptive PID(PI) controller is expected, since the adaptive PID(PI) controller is capable of adjusting its controller settings

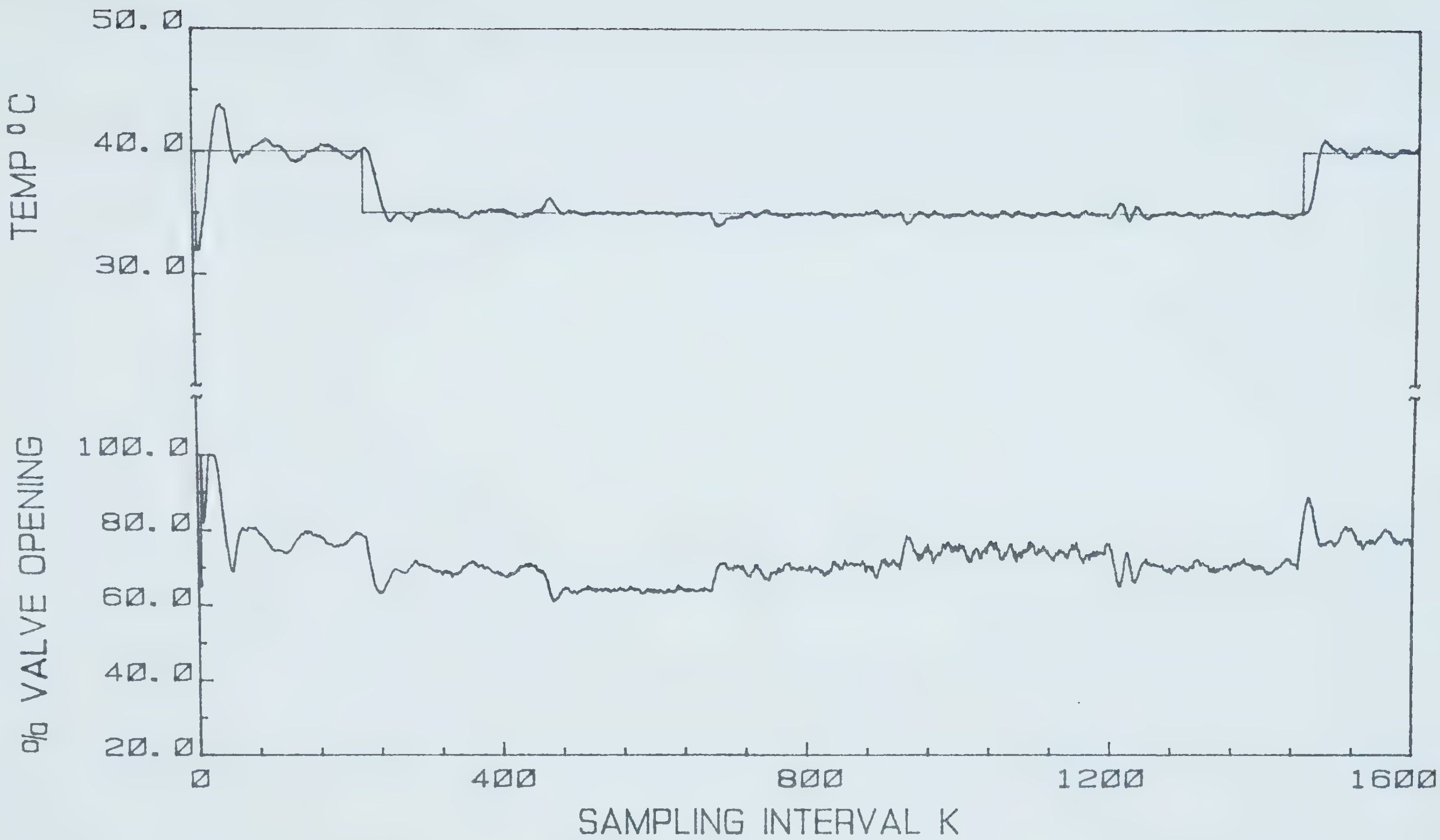

Figure 5.25. Stirred-tank heater response to step disturbances using adaptive PI controller w.o.ffd. (De=3/Da=4/W=0.9/So=10)

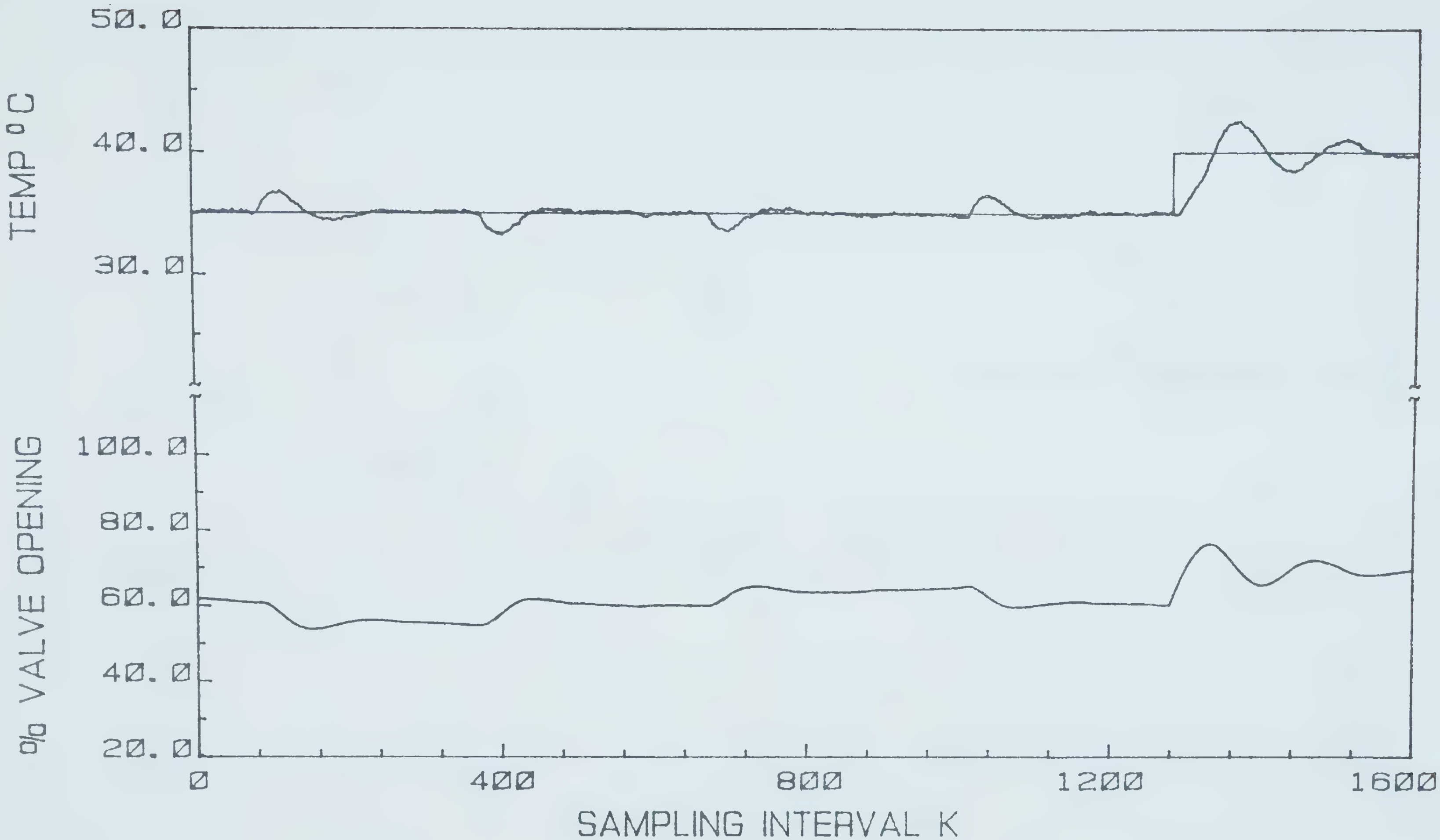

Figure 5.26. Stirred-tank heater response to step disturbances using fixed gain PID (Da=4/Kc=5/Ki=0.005/Td=10)

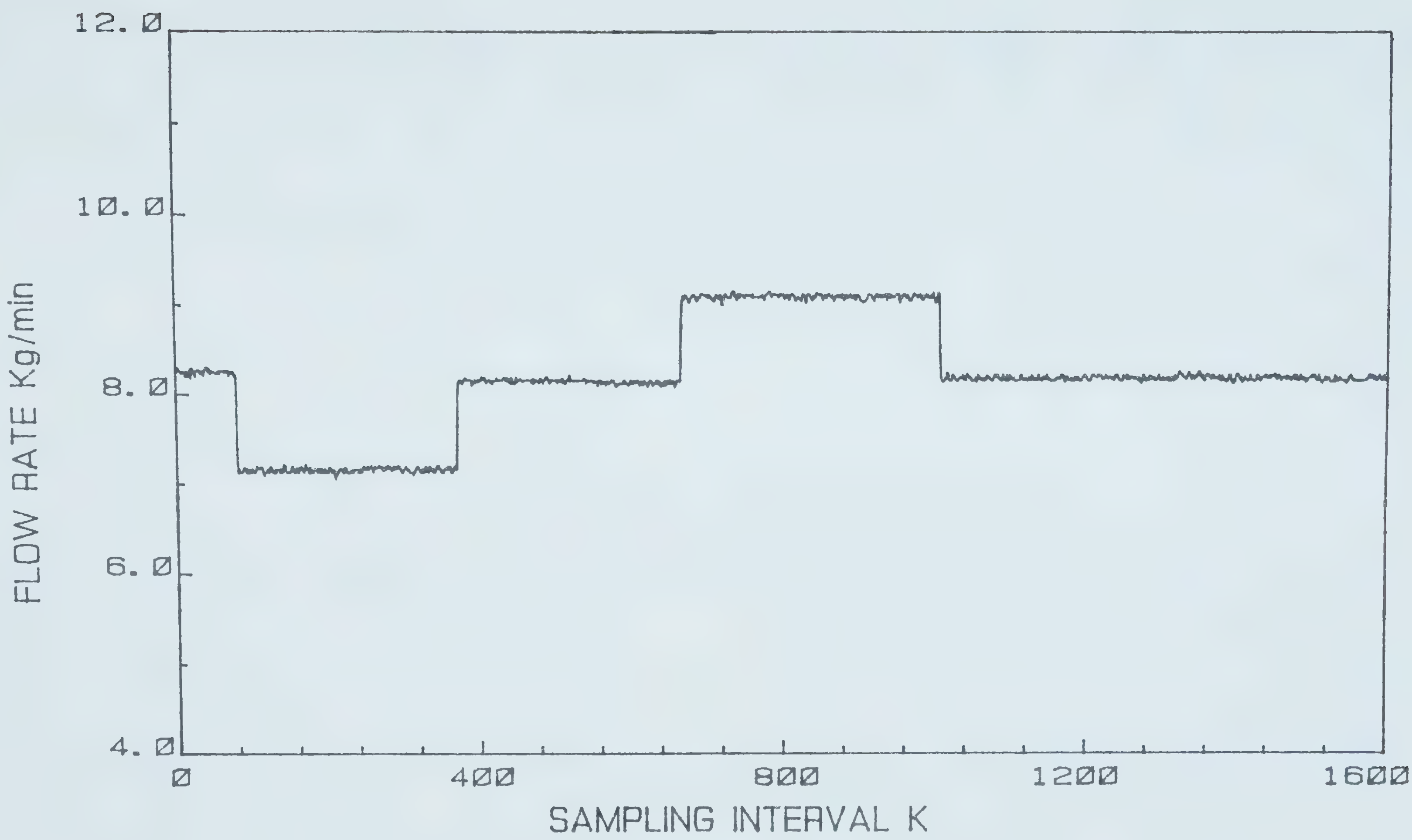

Figure 5.27. Step disturbances introduced to the stirred-tank heater using fixed gain PID (Da=4/Kc=5/Ki=0.005/Td=10)

to both servo and regulatory control situations. Though the IAE technique [Miller, Lopez, Smith and Murrill, 1967] has different formulae to tune the fixed gain PID controller for servo and regulatory controls, the fixed gain PID controller used in all the experimental runs here was fine-tuned to accomodate for servo and regulatory controls as well as system non-linearity.

When the load disturbances are measurable, an adaptive feedforward compensator can be included in the adaptive PID controller. To implement this algorithm, two additional parameters are added to estimate the dynamics of the disturbance. The adaptive feedforward compensator, however, consists of a single gain term as described in Chapter 2.

To evaluate the improvement it provides over the adaptive PID controller alone in the presence of load disturbances, the inlet flowrate measurement was provided to the computer and a similar run to Figure 5.21 was made with the adaptive PID controller plus the adaptive feedforward compensator. The resulting stirred-tank heater response is shown in Figure 5.28 with the step load disturbances shown in Figure 5.29. It is shown that the addition of the adaptive feedforward compensator improves the closed-loop system response considerably. However, it takes a longer period to track a setpoint change after the controller has been tuned in the regulatory control situation. Since the feedforward compensator is designed to improve regulatory control in the presence of a measurable disturbance, a

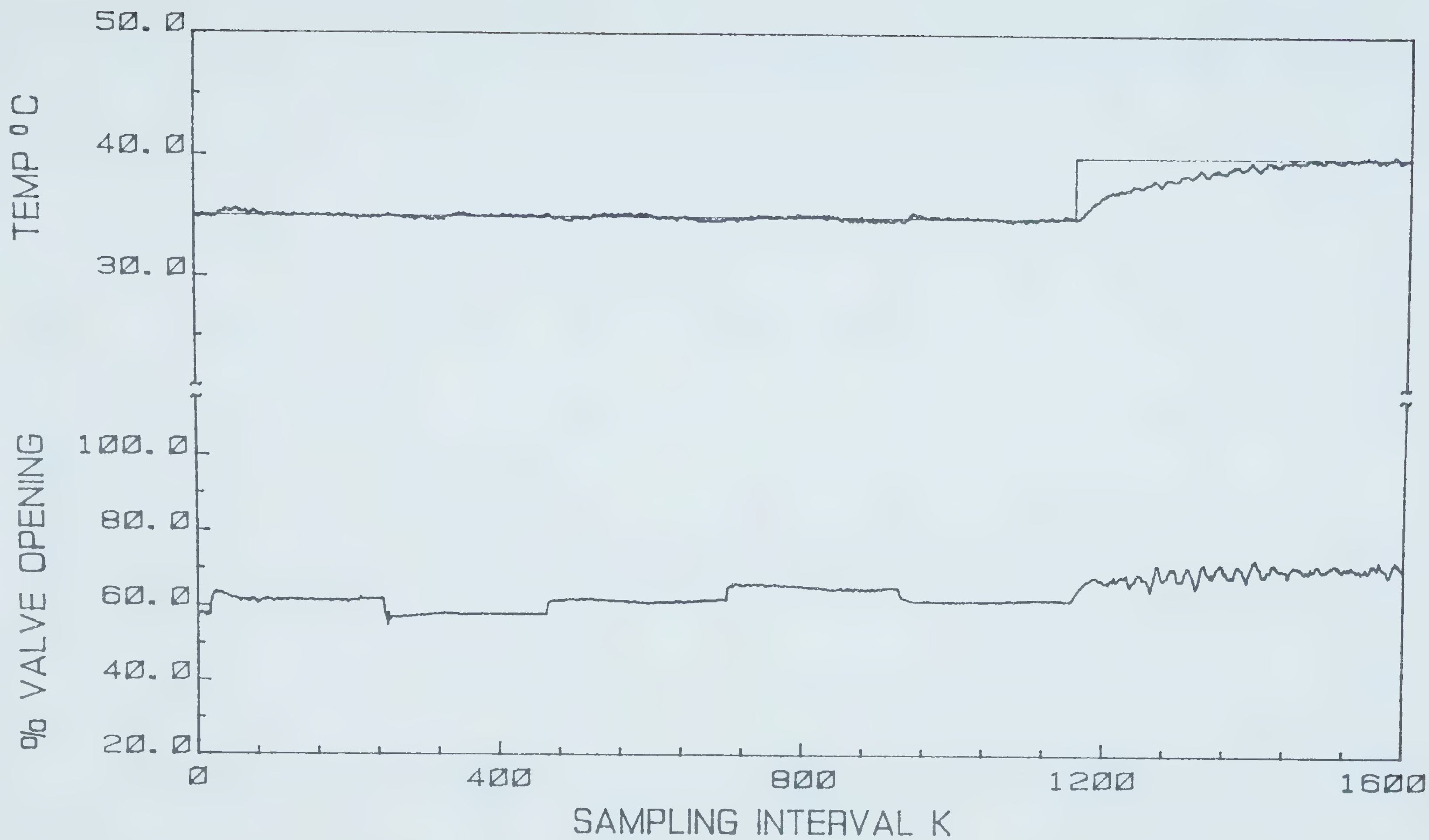

Figure 5.28. Stirred-tank heater response to step disturbances using adaptive PID controller w.ffd. (De=3/Da=4/W=0.9/So=10)

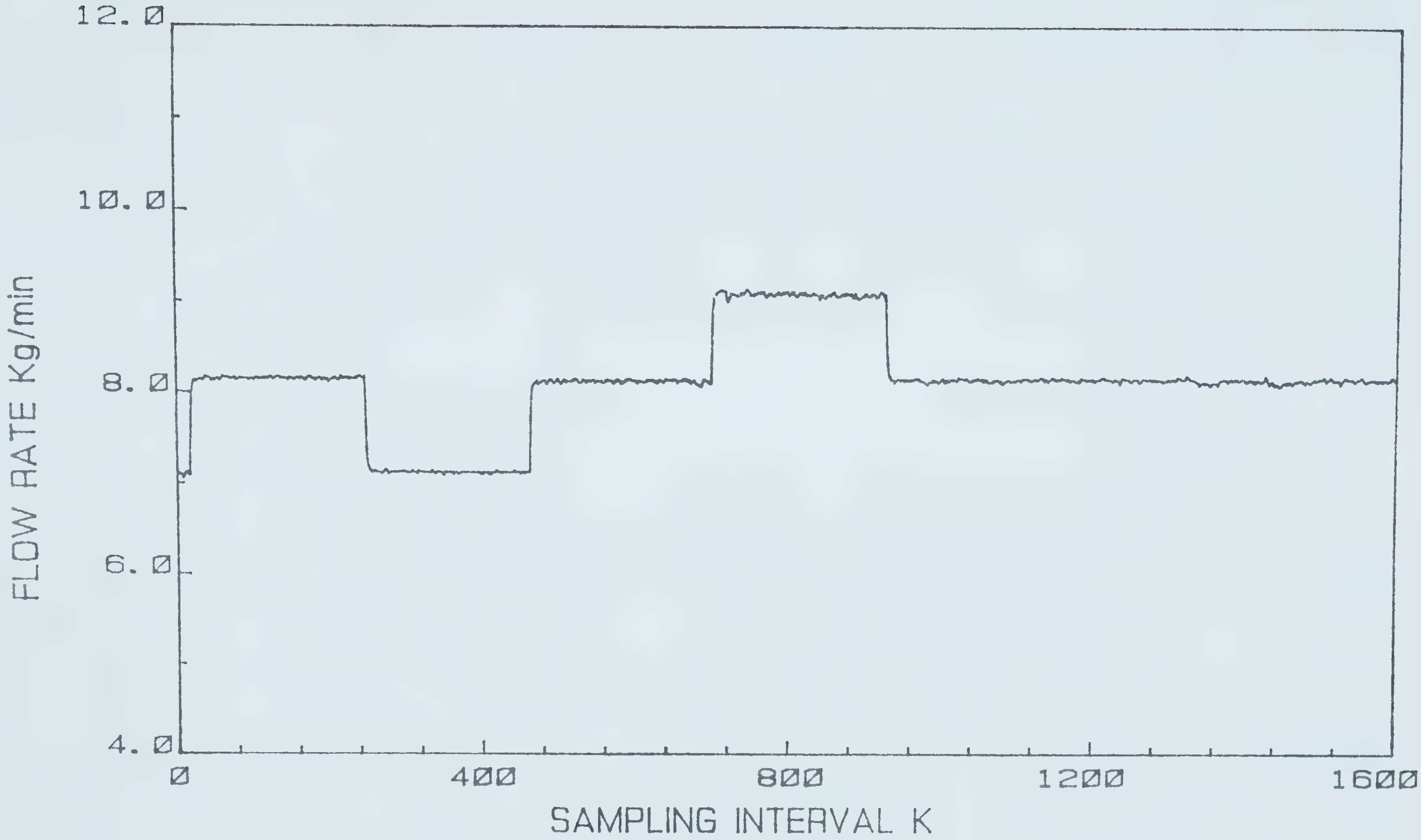

Figure 5.29. Step disturbances introduced to the stirred-tank heater using adaptive PID w.ffd. (De=3/Da=4/W=0.9/So=10)

retuning period is required for the first encounter of a setpoint change.

Figure 5.30 shows the stirred-tank heater response for a 2.5 hour run in which there is only one setpoint change during the initial period, i.e. after being switched from the fixed gain PI controller to the adaptive PID controller. This run is conducted to see the effect of long steady-state operation on the parameter estimates and hence the controller settings. The adaptive PID controller settings are shown in Figure 5.31a, 5.31b and 5.31c.

When the parameter estimates have converged to a set of values which minimize the *a posteriori* estimation error, variations in the forgetting factor value should be minimal. Theoretically, the forgetting factor should converge to a value of 1 at steady state (equation 3.19). Small variations in the forgetting factor, however, do occur and could be caused by the presence of the system noise. This is shown in Figure 5.32. At steady state, the term $(\psi^t(k-1)P(k-1)\psi(k-1))$ in equation (3.19) approaches zero. Equation (3.19) is thus represented by:

$$\mu(k) = 1 - \frac{\hat{e}^2(k)}{\Sigma_0} \tag{5.5}$$

Since the value of Σ_0 is prespecified, the value of the forgetting factor μ is solely dependent on the *a posteriori* estimation error. Even when the parameter estimates have

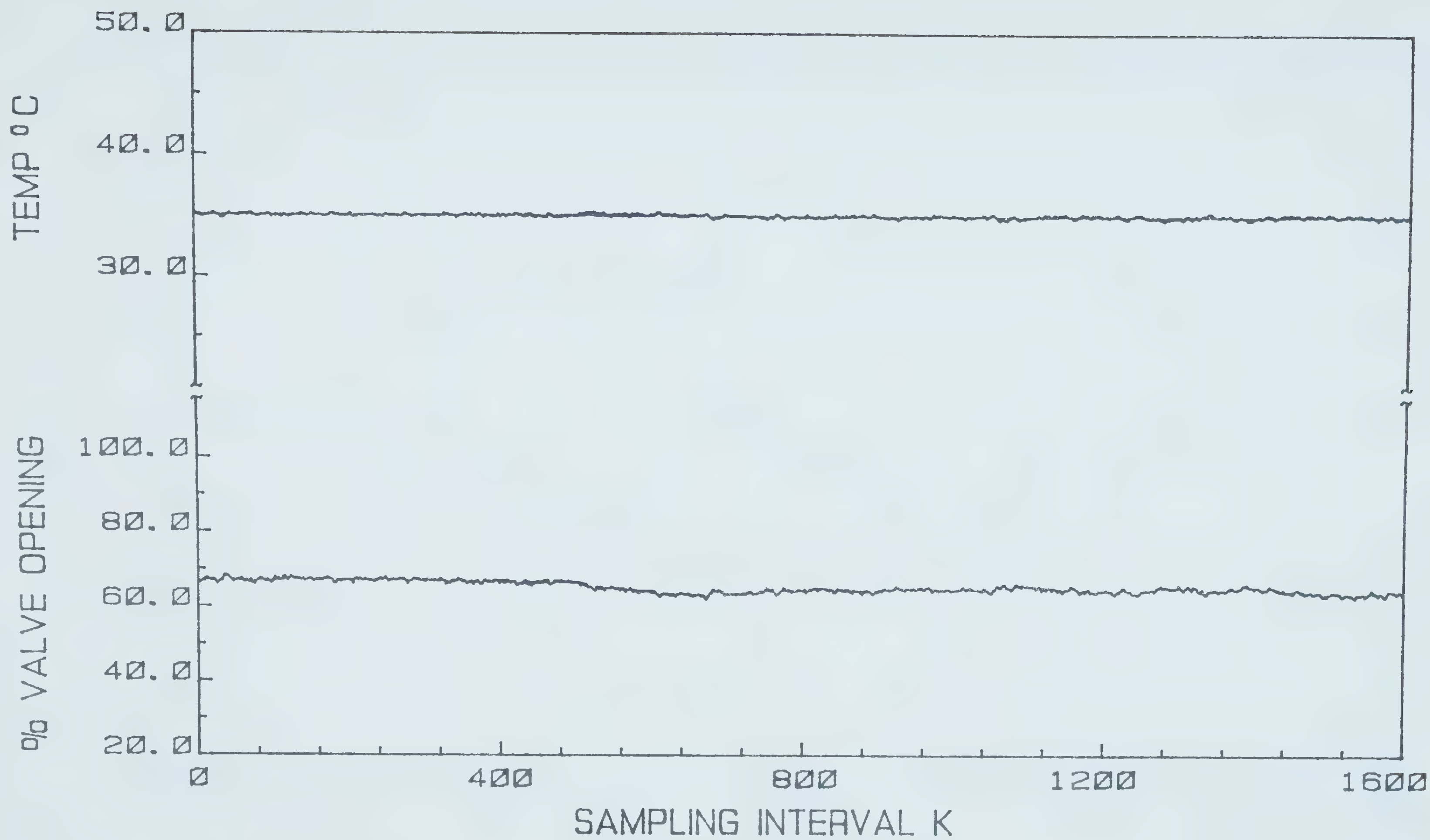

Figure 5.30. Stirred-tank heater response using adaptive PID controller for 2 hours w.o. setpoint change (De=3/Da=4/W=0.9/So=5)

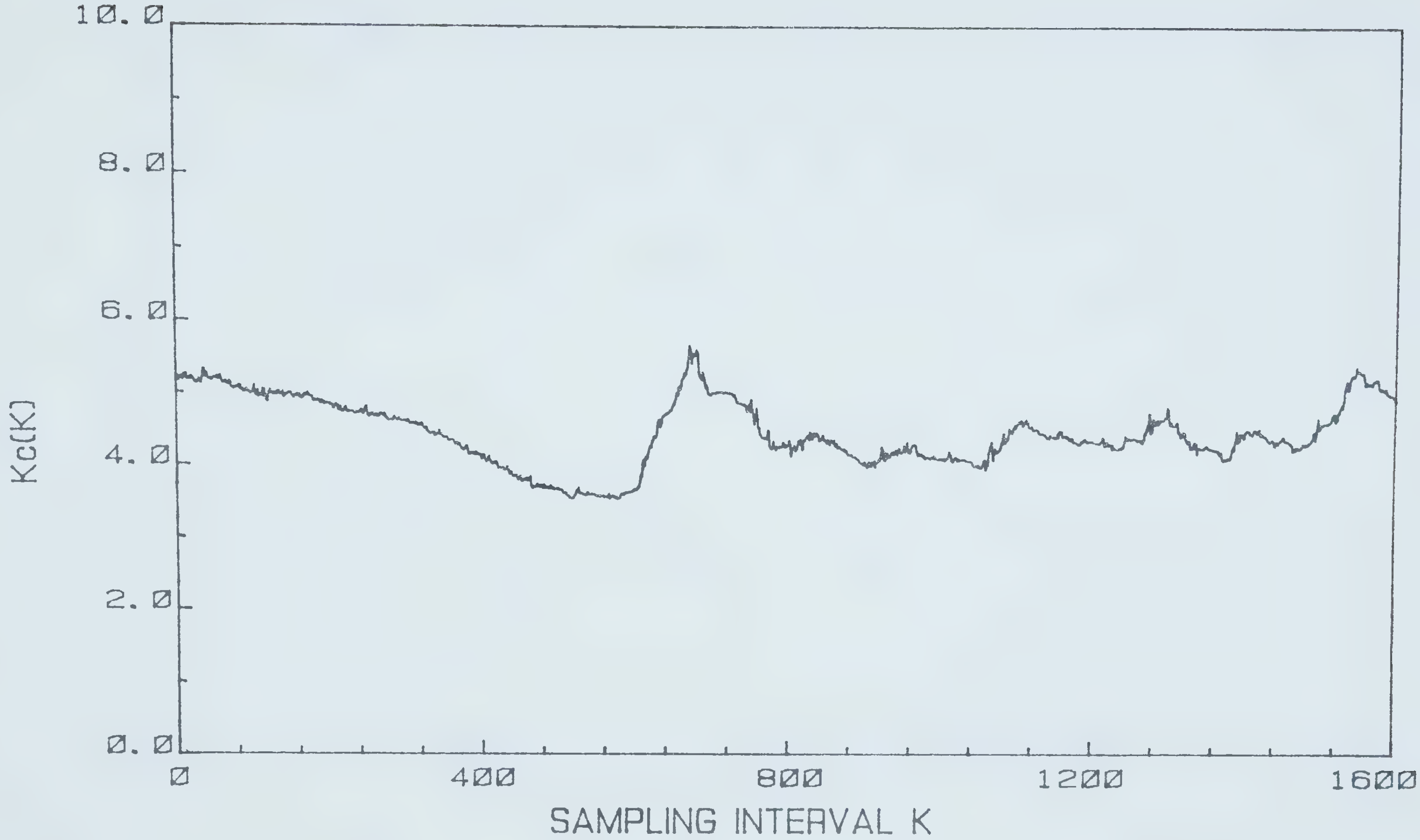

Figure 5.31a. Controller settings of adaptive PID controller (De=3/Da=4/W=0.9/So=5)

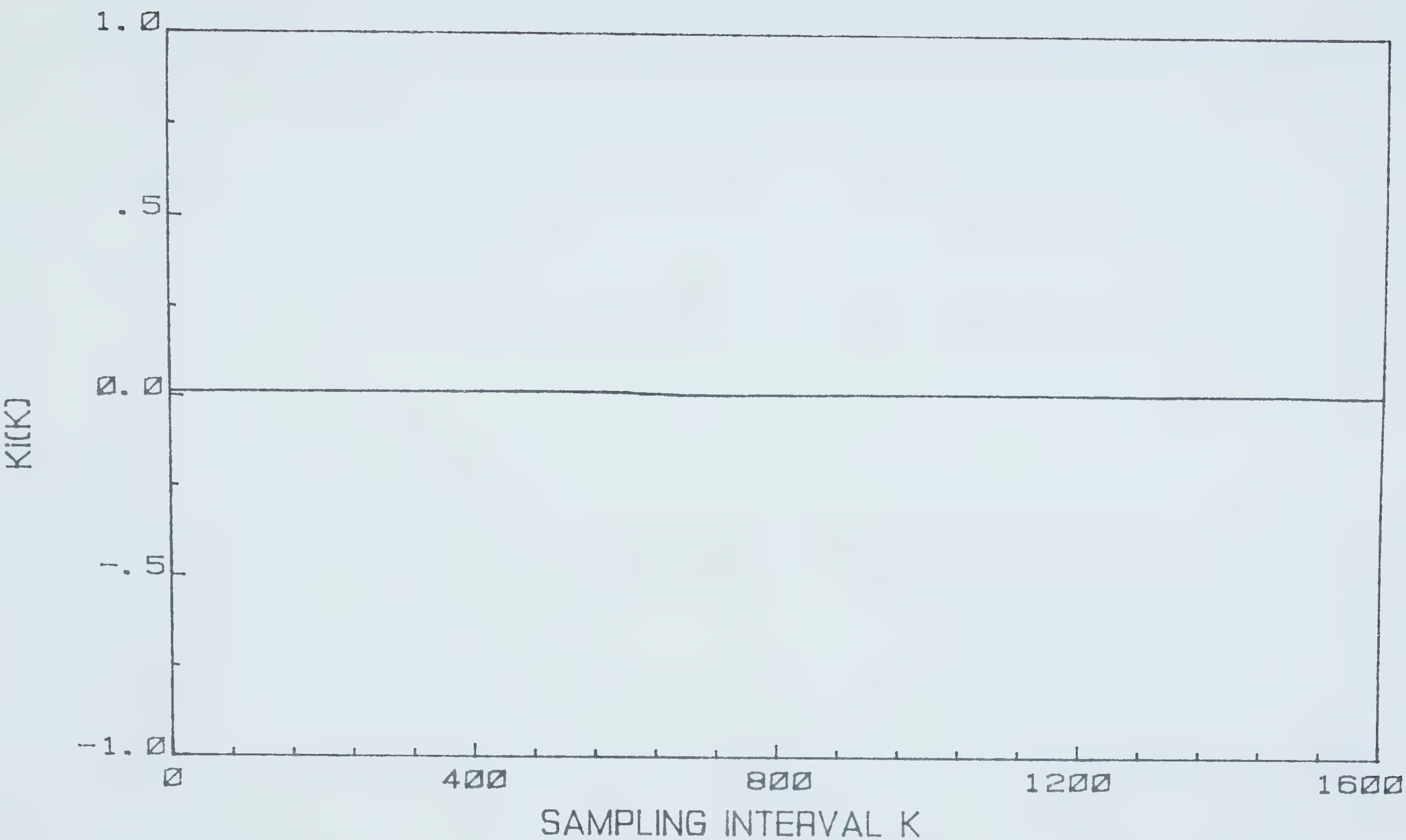

Figure 5.31b. Controller settings of adaptive PID controller
(De=3/Da=4/W=0.9/So=5)

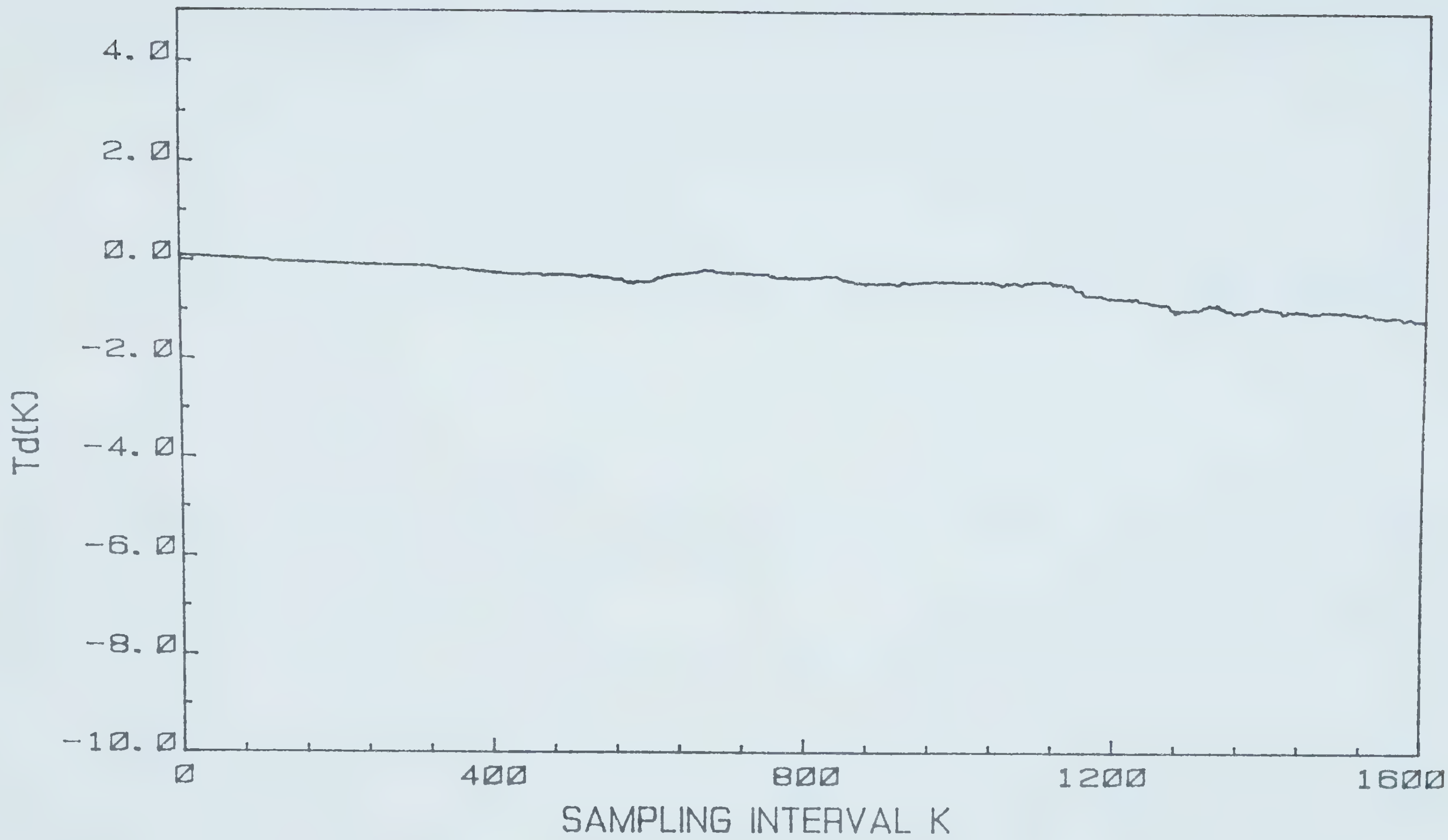

Figure 5.31c. Controller settings of adaptive PID controller (De=3/Da=4/W=0.9/So=5)

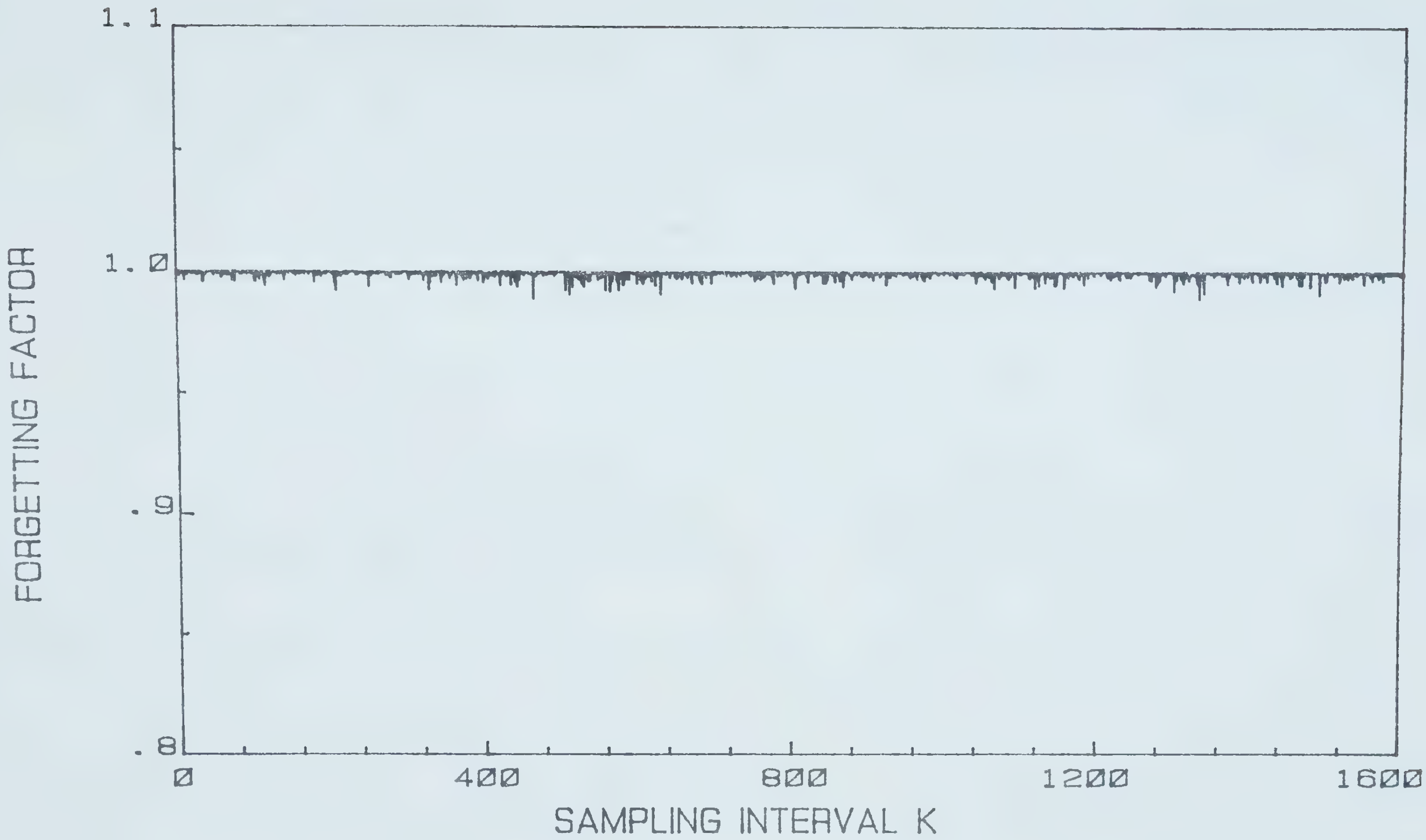

Figure 5.32. Forgetting factor of adaptive PID controller
(De=3/Da=4/W=0.9/So=5)

converged to a set of values, if the estimations are corrupted by noise, the *a posteriori* estimation error will be small but not zero. This could also be caused by the accumulated roundoff error of the finite word length computer. The accuracy of the 16-bit words HP-1000 digital computer for single precision is 6 significant digits. Therefore, the value of the forgetting factor varies slightly even at steady state. These small variations in the forgetting factor further leads to small changes in the controller settings. Variations in Kc can be seen in Figure 5.31a, but the value of Kc is bounded within a value of 4 to 5.

5.4 Summary

This chapter describes the stirred-tank heater and control hardware that were used to experimentally evaluate the adaptive PID and PI control algorithms. It also presents and compares the results obtained by using the adaptive PID(PI) controller with the conventional, fixed gain, discrete PID controller. The adaptive PID(PI) controller is shown to outperform the fixed gain PID controller in all cases, i.e. for the case of a known and constant time delay, an unknown but constant time delay and a varying time delay. Moreover, the adaptive PID(PI) controller ensures closed-loop asymptotic tracking and regulation even in the presence of finite and unmeasurable sustained load disturbances. The initial choice of the covariance matrix is

found to be non-critical. The pole location w_1 gives the desired small overshoot when it is initialized to a higher value, e.g. w_1=0.8 or 0.9. The initial value of Σ_0, though not sensitive, should be chosen high enough such that the forgetting factor would not remain at the lower limit. From this study, it is also found that the value of the forgetting factor during the commissioning period is a good indication as to which direction the value of Σ_0 should be changed to, e.g. increase Σ_0 when the value of forgetting factor remains near the lower limit or when it is fluctuating wildly during the steady-state operation (cf. Figure 5.5). The addition of an adaptive feedforward compensator improves not only the output response but also smooths the control actions, at the expense of two additional parameter estimates. The adaptive PID(PI) controller can be started with the initial parameter estimation done in the background, while the process is controlled by a fixed gain controller. Knowledge of the initial process parameters is therefore unnecessary, and the adaptive PID(PI) control algorithm can be either used as an adaptive controller or as a tuning algorithm for the fixed gain PID(PI) controller. It is also clear then, that the adaptive PID(PI) controller can replace the existing fixed gain PID(PI) controller in industry, especially when retuning of the controller settings is frequently needed.

6. Conclusions and Recommendations

6.1 Conclusions

An adaptive PID(PI) controller has been successfully developed, implemented and evaluated. The resulting controller has the following properties:

- It is structurally and mathematically equivalent to the conventional discrete PID(PI) controller.
- It is robust and ensures asymptotic closed-loop tracking and regulation even in the presence of sustained unmeasured load disturbances, setpoint changes and/or modeling errors.
- For measurable disturbances, it can be used with an adaptive feedforward compensator.
- It uses the computationally efficient and numerically stable U-D factorization algorithm in combination with a variable forgetting factor to perform the parameter estimation. Simulation and experimental results show the excellent performance of this combination.
- The use of the variable forgetting factor allows tracking of slowly time-varying parameters, and avoids blow-up of the covariance matrix during long steady-state operation. In addition, it provides an on-line tuning parameter, Σ_0, to control the speed of adaptation.
- The algorithm has another on-line tuning

parameter, w_1, to place the desired closed-loop pole, and hence allows the operator to 'shape-up' the desired output response.

The performance of the adaptive PID(PI) controller is first evaluated through some simulation runs. Experimental studies, however, are essential as a follow-up for two major reasons: they can further justify the simulation conclusions; they often lead to new insights and thus complete the entire evaluation procedure. For instance, all adaptive controllers require some kind of initial parameters before they can be operated. From the simulation runs, it is found that the initial parameter estimates could be initialized to zero except for one $\hat{b}$ parameter. The experimental studies, however, reveal that this could lead to undesired large variations in the initial closed-loop system response and might even lead to unstable response. Usually the initial parameters can be obtained either analytically, by simulation, by *a priori* experimental studies or by experience. For the adaptive PID(PI) controller, the experimental studies found that the initialization method in the simulation runs could still be used if background estimation was performed during the initial period while the process was controlled by a fixed gain PID(PI) controller.

The application of the adaptive PID(PI) controller to the stirred-tank heater shows the superior performance of the adaptive PID(PI) controller. The adaptive PID(PI)

controller is therefore a logical candidate for use in industrial processes where conventional discrete PID(PI) control is being used, in particular when retuning of the controller settings is frequently needed. In addition to its use as a 'stand-alone' adaptive controller, the adaptive PID(PI) control algorithm can also be used as a retuning algorithm for the existing conventional discrete PID(PI) controllers.

Despite the amount of research activity in the adaptive control area, the number of applications of adaptive control in industrial processes is still very few. One of the reasons is due to insufficient insight and thus the unwillingness to risk the expensive plants by applying modern control techniques. Derivation of the adaptive PID(PI) controller reveals two significant points, which should help to fill this void:

- Compared to fixed gain PID controllers, adaptive controllers are sophisticated. However, they can be interpreted in a fundamental classical linear feedback control theory framework. In its simplest form, the adaptive controller proposed here has been shown in this work to be structurally and mathematically equivalent to a conventional discrete PID(PI) controller.
- Though an adaptive controller is capable of learning the process and adjusting its controller settings automatically, it should not

be regarded as a 'magic blackbox' and should be used properly and accordingly.

The software developed to implement computer control of the stirred-tank heater uses the multiprogram feature supported by HP-1000 computer system. This feature allows the operator to make any changes through the operator console without suspending the control cycle. It is easy to use and modify for experimental evaluations of any adaptive and/or fixed gain control algorithm.

6.2 Recommendations

The following recommendations regarding future work can be made:

- Further experimental evaluations of the adaptive PID(PI) controller on time-varying systems and/or non-minimum phase systems should be performed. The controller should also be applied to multivariable system by configuring it as multi-loop controller.
- To provide easiness of industrial testing, the adaptive PID(PI) control algorithm should be implemented on a portable microprocessor-based computer.
- Stability and convergence analysis on the adaptive PID(PI) control algorithm.
- Theoretical investigation on robustness issues, for example additional $\hat{b}$ parameters to handle

unknown and/or varying time delay systems.

- Since a scalar variable forgetting factor represents only a measure of the *total* information content in the estimator, one has no control on how this *total* information is distributed among the various parameters. Therefore, the possibility of using a vector of variable forgetting factors to control this information distribution should be investigated.

Apart from those stated above, improvements on the stirred-tank heater are also recommended. These include:

- Installation of a flow controller on the inlet cold water line. This is mainly to avoid sudden large variations on the inlet flowrate when other laboratory equipments are in use.
- Replacement of the proportional level controller by a PI controller to eliminate the existing offset.
- Expansion of the range of the temperature transmitter to allow for a wider operating range. This is due to the large variations in the cold water temperature from the winter months to the summer season.

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